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REINFORCED CONCRETE COLUMNS

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To Be Presented February 14th, 1923

Synopsis

The purpose of this paper is primarily to determine, by scientific methods, values of safe working stresses for the several types of reinforced concrete columns, and to indicate those excessive and, therefore, unsafe stresses permitted by the 1917 and 1921 Specifications suggested by the Joint Committees on Concrete and Reinforced Concrete† and on Standard Specifications for Concrete and Reinforced Concrete,‡ respectively, particularly the excessive and dangerous stresses permitted by the 1921 Specifications for spirally reinforced columns.§

In Section I, the effect of the non-homogeneity of concrete on its various constants of stress and elasticity is analyzed. An analysis based on the test data and observations of all appropriate columns tested in the United States, in which the effect of the variation of the ultimate strength of the concrete is mathematically eliminated, leads to equations expressing the ultimate strength of the several types of reinforced concrete columns in terms of all the column variables. The analysis gives a clear insight into and establishes the functioning of the several column components and the strength to be expected from them *per se*. One important result attained is the knowledge of the uniformity of strength developed by longitudinal reinforcing rods per unit area.

In Section II, the mathematical equations for the spirally reinforced concrete column are developed. By the substitution of known values for the variables of these equations, simultaneous values of longitudinal column core stress and spiral stress are obtained, that check observation values within experimental measure, thus verifying the equations. It can be shown by these equations that:

1.—The ultimate strength of spirally reinforced concrete is independent of the yield-point strength and the ultimate strength (tensile) of the spiral reinforcement.

Note.—These papers are issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on this paper will be closed in June, 1923, and when finally closed, the paper, with discussion in full, will be published in *Transactions*.

* Pittsburgh, Pa.

† *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1101.

‡ *Proceedings*, Am. Soc. C. E., August, 1921, p. 59.

§ The Specifications submitted by the Joint Committees on Concrete and Reinforced Concrete and on Standard Specifications for Concrete and Reinforced Concrete will be referred to in this paper as the "1917 Specifications" and the "1921 Specifications," respectively.

2.—The spirally reinforced concrete column develops maximum strength and fails when the spiral is stressed at most to the yield point. The failure of the spiral in tension by further increase in column compression strain is a relatively unimportant phenomenon.

3.—It is impossible to stress the steel spiral beyond the yield point by compression on the concrete core of the column.

In Section III, the concept of reliability is analyzed. Factors governing reliability are critically examined, and a method of accurately expressing it in a reliability number determined by the well known error function is developed. The reliability number is applied to substantiate the equations of Section I and is also a necessary factor in the determination of the safe working stresses in Section IV.

In Section IV, the factors essential in the determination of safe working stresses in general are enumerated. Formulas are derived mathematically without the inclusion of the "guesswork" factor of safety or its equivalent, for the determination of safe stresses for the several types of reinforced concrete columns. Stresses calculated from these formulas for many examples within the range of possible selection of column variables are tabulated in comparison with those permitted by the 1917 and 1921 Specifications. By comparison, the stresses permitted by these specifications are shown to be inconsistent and, in many cases, unsafe.

SECTION I.—EMPIRICAL FORMULAS*

1.—Introduction

Because of the relatively large expense and the necessity of special equipment, only a small number of plain and reinforced concrete columns have been tested. In each series of tests the number of specimens, especially of duplicate specimens, has been less than desirable in order to determine definitely the effect of each variable affecting the strength of the column.

This Section is devoted to an analysis, as one large group, of the various important groups of columns that have been tested, thereby eliminating, not only the variation in results due to factors both unknown and incapable of being measured, but also the personal equation of each investigator. By thus increasing the range of variables and the number of test specimens, a surer and wider insight into the functions of the reinforcing components and a more exact derivation of empirical formulas are obtained. The formulas developed in this Section, besides being of direct benefit in the determination of the probable strength of a compound column, are applied in Section IV in the determination of safe working stresses. No common dimensions of the columns, such as length, diameter, slenderness ratio, strength of concrete, percentage of reinforcement, etc., are found in the several groups of tests. It is a difficult task, therefore, to co-ordinate all the tests and from them to deter-

* An empirical formula is one derived directly from an experimental basis, assumed to be resolvable into simpler laws, and is only to be accepted as true for values lying within the limits of the observed quantities.

mine the effect of each variable and to formulate a general law for the strength of the columns of each type. Evidently columns having concrete of different strengths and different percentages of spiral and longitudinal reinforcement, cannot be directly compared. Some means of comparison, therefore, must be found, in order to derive the benefit of including every column in the mathematical analysis. A different method of analysis must of necessity be adopted for each type of column.

2.—Importance of the Empirical Formula

The formulas in this paper are derived from an almost purely empirical basis. Such formulas are not generally held in highest esteem, but they should not, especially in this case, be considered of little importance. One important branch of mathematics, the Theory of Errors, deals with its subject-matter empirically. Scientifically, there is no such thing as an error. The "error" or discrepancy from a predicted result or from other observations is due to an accumulation of minutiæ or influences, of which knowledge is limited or lacking, or of influences too complicated to permit an attempt at a mathematical solution. Empirical formulas give no insight into underlying operating causes. Newton considered a "force" as an operating cause and a concrete thing in itself; modern science has shown that a "force" is merely a means of measuring change in motion, cannot be measured by any other unrelated means, and is incapable of being separated from matter.

Science has changed its attitude of endeavoring to determine the reason for phenomena to one of determining the manner in which the phenomena occur; it is a change of the manner of inquiry from formerly asking why to the present inquiry of how.

The engineer is most concerned with results. The empirical formula gives results accounting for the effects of all actuating influences without in any way attempting to analyze and separate the effect of each, and directly discounting the effect of the accumulation of minutiæ and unknown influences.

3.—Types of Columns

The types of columns included in this analysis are:

1.—Structural steel.

2.—Cast iron.

3.—Concrete, unreinforced (permissible up to six diameters).

4.—Concrete, reinforced:

(*a*) With longitudinal steel rods, wire laterals binding the rods at intervals about equal to the column diameter.

(*b*) With structural steel elements, these elements themselves forming a column. The steel structure as a whole, columns, floor-beams, etc., is first erected, the concrete being cast in place in forms built around the steel structure.

(*c*) With wire spiral. This type is not used in practice, building codes requiring longitudinal steel rods as reinforcement in addition to the spiral.

(*d*) With spiral and longitudinal steel rods.

(*e*) von Emperger type: A spiral and rod reinforced concrete column in the center of which is a cast-iron tube or column. This type was originated by Dr. von Emperger, of Austria.

The structural steel and cast-iron columns are included for comparison because in most cases they can be substituted for the reinforced concrete column and *vice versa.* The plain concrete and spiral, without rod, reinforced columns are included to develop the formulas for the strength of the rod reinforced and the spiral and rod reinforced columns. Timber, stone, and brick columns, or rather piers,* not in the same category with the reinforced concrete column, are omitted.

4.—Comparison Equations

Formulas giving the reduction in strength with length for different types of columns are in common usage. These formulas, however, do not afford a means of comparing the relative reduction in strength of different types of columns. In order to afford such a comparison, Equations (1) and (2) were developed. Although a square steel column cannot strictly be compared with a round one, by substituting as the diameter the distance from face to face of the steel column, in Equations (1) and (2), satisfactory results are obtained.

For built-up steel columns or cast-iron columns of one cross-sectional area the reduction in strength with length is not uniform for all conceivable distribution of this area. The usual variation of this distribution, however, is not great, owing to the limited possible methods of fabrication, etc. The relation between ultimate strength and length of column is shown in Equation (1), as follows:

$$U_0 = K_1 - K_2 \frac{L}{D} \quad \text{..........................} (1)$$

in which,

U_0 = ultimate strength of the column;
L = length of the column;
D = diameter of the column; and
K_1 and K_2 are constants.

Equation (1) can be reduced to:

$$\frac{U_0}{K} = \frac{K_1}{K_1} - \frac{K_2}{K_1} \frac{L}{D}$$

or,

$$U = 1 - K \frac{L}{D} \quad \text{..........................} (2)$$

Equation (2) furnishes a means of comparing the relative reduction in strength with length, for various types of columns, by means of the constant, *K*, for each type.

* In *Technologic Paper No. 111,* U. S. Bureau of Standards, Mr. J. G. Bragg gives the results of a comprehensive series of tests on brick piers, by far the largest ever tested, and also a résumé of the only other important series of tests that have been made on brick piers

5.—Unreinforced Concrete

A.—General

Concrete is exceedingly non-homogeneous, and, therefore, its properties such as strength, rigidity, etc., vary throughout its mass. Because of this non-homogeneity, the various constants, such as ultimate stress, Poisson's ratio, etc., for concrete are difficult to determine. As the constants vary when all known and controllable operating factors are made constant, not only must the average value of the constant be determined, but also the manner of its variation, that is, a number that measures a quantity corresponding to the probable error.

There is no means of accurately determining the strength, even the average strength, that concrete made from a hitherto unused aggregate will develop except by means of test specimens, made in the same manner, the same proportions, percentage of water, etc., as the concrete to be used in the final structure itself.* Steel made under rigid supervision and qualifying for certain definite strength tests, is received as a finished product at the point of erection and is erected with the positive knowledge of its possessing a certain definite strength.

The actual strength of the concrete in a structure is different. Such strength cannot be determined until the concrete has been placed and test cylinders from the same mix as that of the structure have been tested.

B.—Test Specimens

Concrete test specimens are made in two standard forms: The cube, usually 6 in. along an edge, and the cylinder, two diameters in height, usually 8 by 16 in., or 6 by 12 in. The test cylinder gives directly the ultimate strength of a column made from the concrete, corrected for loss in strength due to length.

Arthur N. Talbot, Past-President, Am. Soc. C. E., states:† "The strength of columns and cylinders agrees fairly well." The strength of cylinders and columns agrees only "fairly well" owing to the natural variation in the strength of the concrete itself, irrespective of its external form. Two columns, or two cylinders, made from the same batch of concrete, will not be of the same strength, and for the same reason, a column and a cylinder made from the same batch, except by chance, will not be of the same strength.

The cube develops considerably more strength than the cylinder, but is a form more difficult to mould properly, requires more skill to make, and gives less reliable results. From groups of auxiliary test specimens, made to determine the strength of concrete in columns,† Professor Talbot has shown experimentally the greater strength of cubes. These results show the strength of the cube to be 1.5 times that of the cylinder. The ratio of the strengths, however, must be dependent on the relative size of the test specimen and the

* Complete fundamental information on the preparation and properties of concrete is contained in *Technologic Paper No. 58,* U. S. Bureau of Standards.

† *Bulletin No. 20,* Univ. of Illinois Eng. Experiment Station.

coarse aggregate, hence there can be no fixed ratio between the strength of the concrete in the cylinder and the cube.

The greater strength of the cube as compared to that of the cylinder has a theoretical basis and is the result of the method of compression failure of concrete. Concrete in compression fails in shear in a plane or planes inclined about 35° to the axis of compression. The concrete column and the test cylinder of a length equal to twice the diameter permit this shear to occur freely. The form of the cube, however, forces the shear failure to occur in a plane inclined at an angle of 45° or more and, therefore, increases the compressive force necessary to cause this shear and concomitant failure. This method of failure must be understood for the further analysis of reinforced concrete elements.

There is always the danger that the layman or even the ordinary supervisor of tests will not appreciate the fact that the ultimate stress in the cube is much higher than that which a column of the same material is capable of developing, and, from the results of cube tests, will consider the concrete much stronger than it actually is.

The late J. B. Johnson, M. Am. Soc. C. E., states* that "the cube is not the best form" of test specimen, because of the discrepancy between column and cube ultimate stresses, but he did not explain the reason for this difference. Professor Talbot has also stated† that it is probable that many engineers have been misled by high values obtained on test cubes. On account of the risk just mentioned, the cube should be discarded as a test specimen for determining the merit of concrete for compression members such as columns. For highway work, etc., it is possible that it may be a better test specimen than the cylinder.

C.—Constants for Concrete

Ultimate Stress.—Concrete being a non-homogeneous material, its compressive strength varies along the compression axis of a concrete column. A column under increasing compression will fail at the weakest point in the axis, and the ultimate strength of the concrete, determined in this manner, is not its average ultimate strength, but is an accurate measure of the weakest concrete in the column. Therefore, the longer the axis, the greater the length in which failure can occur, the lower the average strength, and the less the variation in strength. This may be the only reason for the reduction in strength with column length. Test cylinders in compression must fail within a comparatively limited length, and for this reason alone the test cylinder, therefore, should show a higher average compressive strength than the column, but a greater variation in strength. This greater variation in strength is known; but comparatively, for the length, the test cylinder is weaker than is otherwise indicated. This lesser strength is probably due to the relative size of the largest aggregate and the diameter of the test specimen for the cylinder and the column.

* "Materials of Construction," Paragraph 113, 1919 Edition.

† *Bulletin No. 10,* Univ. of Illinois Eng. Experiment Station.

Ultimate Strain.—The ultimate strain of concrete is subject to variations similar to those in the ultimate stress, and it will vary when every variable is controlled and made constant. The ultimate strain is of use only in its relationship to the ultimate stress, and is to be determined as a function of that stress, if it is one. For various specimens of one strength, it is not to be expected that the ultimate strain will also be equal or constant, nor that the ultimate strains and stresses bear an exact relationship for a group of specimens made from one batch of concrete. The strain measurements of a column or test specimen are only an average of the strains along the compression axis for the length over which these strains are measured. The ultimate strain determined in this manner is not the true ultimate, nor the strain in the proximity of the point of failure. The strain measurements on test cylinders, as in the case of those of ultimate stress, will be nearer the actual quantities. By extrapolation of the stress-strain curves for concretes of various strengths,* the three average values of ultimate strain shown by the points in Fig. 1 were obtained. The representative line for these points shows marked characteristics and an evident relationship between concrete strength and ultimate strain.

These values, obtained by extrapolation, are roughly checked from observation of the strain at which the shell of a spirally reinforced column commences to scale. Although this scaling would indicate the ultimate strain of unreinforced concrete, such concrete frequently spalls and gives indications of failure prior to ultimate stress. These indications, together with the difficulty of observing the exact stress and strain at which initial spalling occurs, make results obtained from this method less accurate than are to be desired.

Some results seem to refute the curve shown on Fig. 1, which indicates that concrete of an ultimate strength of about 600 lb. per sq. in. has an ultimate strain of 0.0006 in. per in. Apparently contradicting this are the results of Professor Talbot's tests of two concrete columns with structural steel reinforcement, the concrete having a strength of 680 lb. per sq. in. Adding this unit strength to that of the steel reinforcement to obtain the total column strength would indicate that before failure the columns were strained to at least 0.0015 in. per in. The conclusions to be drawn from these two columns of concrete of low strength are not certain or final, because some reason other than the ultimate strain of the concrete being greater than 0.0006 in. per in. may have been the cause of the strength added by the concrete. The structural steel columns restrained the concrete inside, and may have added to the strength in a manner similar to that of a spirally reinforced column. This action, however, would not be present in a column reinforced only with rods, as the rods would have no restraining action on the concrete enclosed by them.

Poisson's Ratio.—To measure Poisson's ratio for a non-homogeneous substance such as concrete is difficult. Poisson's ratio is that of the lateral to the longitudinal strain for a particle of the concrete infinitely small, when it can be assumed that the strains are homogeneous throughout the particle.

* From the curves given in *Technologic Paper No. 12*, U. S. Bureau of Standards, *Bulletins Nos. 300* and *466*, Univ. of Wisconsin, and *Bulletin No. 20*, Univ. of Illinois.

Any measurements of the longitudinal or lateral strains in concrete necessarily must be averages, and it is impossible to obtain measurements of these quantities that are averages of the same set of points in the concrete. The longitudinal strain will be an average of the set of adjacent longitudinal points in the length of the column, and the lateral strain will be an average of the set of adjacent lateral points in the column. These two sets of points will have only an infinitesimal volume or only one point in common, and their ratios will be valueless for giving Poisson's ratio. A roughly approximate value of Poisson's ratio can be obtained by taking the average longitudinal strain of the column for several points around the diameter, and twelve or more measurements of the lateral strain in its length. Professors Talbot and Withey have made longitudinal and lateral measurements, but the lateral measurement was taken at only one place on the column. The computation of Poisson's ratio from these measurements leads in many cases to absurd results,

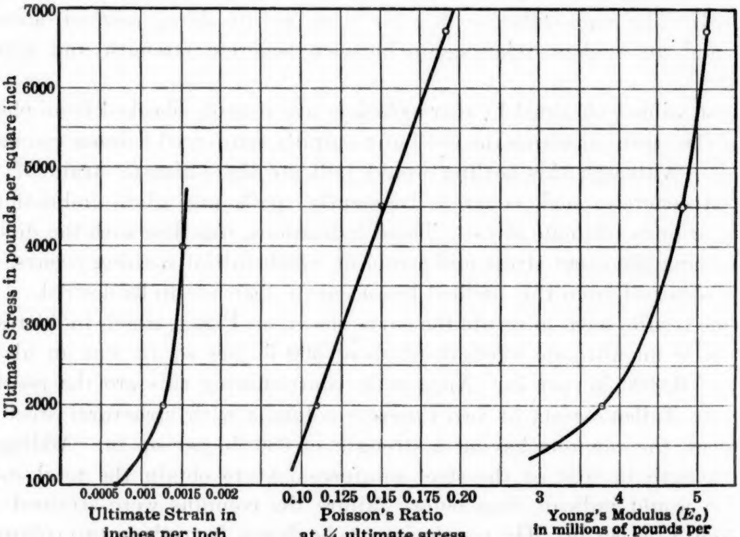

FIG. 1.

the values obtained being frequently greater than 0.5. This is impossible, being a mathematical statement regarding a substance more fluid than a perfect fluid, or a direct contradiction of the principle of the conservation of energy. Fig. 7 shows several curves giving the relation between Poisson's ratio and stress, computed from the measurements of Professors Talbot and Withey. The results have qualitative if not quantitative value. Fig. 1 shows the relation between Poisson's ratio and the ultimate strength.

Young's Modulus.—For concrete, Young's modulus, E_c, is a variable, being a function of the stress. No part of the stress-strain curve of concrete obeys Hooke's law, that is, that E_c is a constant. The values of Young's modulus, computed from the ultimate strain and ultimate stress, are shown on Fig. 1,

with the representative curve, the points representing averaged values of the quantities obtained from the same source as the values for Poisson's ratio.

Yield Point.—There is no yield point of concrete. The concept of yield point was originated for steel, which has a true yield point. The proof that there is no yield point for concrete needs no other evidence than the fact that authorities cannot agree on a method of obtaining it, or arrive at the same value in their determination of it.

The properties of other materials used in a monolithic structure may make it essential that a certain strain is not exceeded, which would place a limit on the "working strain". There is a stress (as in steel), a certain definite percentage of the ultimate stress, an infinite number of repetitions of which will not cause failure, nor too large an increase in strain. These, then, are definite criteria to be observed in concrete, instead of the so-called "yield point."

6.—Unreinforced Concrete Columns

As unreinforced concrete columns are never used in practice, tests of specimens of this type are considered of small value. Test results of only eleven such columns can be found, and these results, together with data on the columns, are given in Table 1.

TABLE 1.—Plain Concrete Columns.

Source.	Diameter, in inches.	Slenderness ratio, $\frac{L}{D}$.	Ultimate Stress, in Pounds per Square Inch.		Ratio: Columns to cylinders.
			Cylinder, average.	Column.	
M.*......	14	4.2	2 084	2 045 2 137 2 087	97.5 102.4 100.1
"........	14	8.5	1 740	1 698 1 836	97.8 105.5
"........	14	16.6	2 272	1 676 1 819	73.6 80.0
W_{466}†	10.5	9.72	2 600	2 660	102.4
"........	"	"	2 400	2 660	111.0
"........	"	"	2 250	2 480	110.0
A. C. I.‡....	20.2	7.2	2 775	2 990	107.9
"........	"	"	2 518	2 500	99.3

* "Tests on Concrete Columns, Plain and Reinforced," at Lehigh Univ., 1915, by F. P. McKibben Assoc. M. Am. Soc. C. E., and A. S. Merrill, *Proceedings*, Am. Concrete Inst., 1916, p. 200.

† "Tests on Concrete Columns," by Prof. Owen Morton Withey, Univ. of Wisconsin, *Bulletin 466*, Eng. Series.

‡ "Comprehensive Tests for the Institute at the Pittsburgh Branch, U. S. Bureau of Standards," *Proceedings*, Am. Concrete Inst., February, 1915.

It is evident that the strength of concrete columns will decrease as the length increases. From a set of columns of the same diameter and strength of concrete, but of varying lengths, this reduction in strength due to increase in length could be obtained directly, either graphically or algebraically. With the set of columns under discussion of concrete of different strengths and different diameters, etc., some other method must be used. Fortunately, test

cylinders were made from the same batches of concrete for all the test columns, and these cylinders form a base from which to measure the strength of the individual columns expressed as a percentage of the strength of the concrete in the test cylinders.

From the slenderness ratio and the ratio of the strength of the column to that of the test cylinder for each column of the group, the relation between them can be determined. These points are shown on Fig. 2. The representative line passes the 100% abscissa at the ordinate 8.3, which signifies that a small test specimen with the ratio, $\frac{L}{D} = 2$, will develop the same ultimate stress as a column 8.3 diameters long. Theoretically, the line should cross the 100% abscissa at the value of $\frac{L}{D} = 2$, as the ratio for the small test specimen has that value. The discrepancy is probably due to the larger diameter of the test column, the unit strength being a function of the relative size of the test specimen and the average size of the largest aggregate. That this is so can be demonstrated by considering the extreme case in which the size of the large aggregate is equal to the diameter of the test specimen. It can readily be seen that such a test specimen would suffer a great reduction in strength due to the large size of the aggregate.

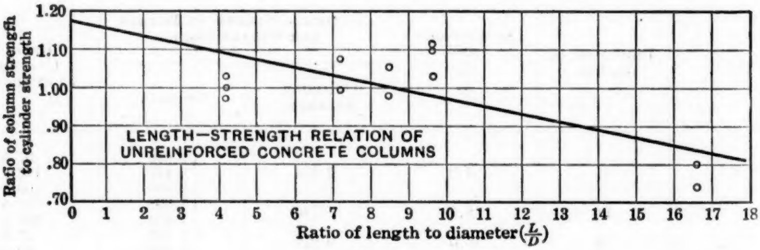

FIG. 2.

The equation of the length-strength line previously obtained, and expressed in Equation (1), is:

$$U_0 = 1.17 - 0.020 \frac{L}{D} \quad \text{(3)}$$

which can be expressed in the form of a reduced equation similar to Equation (2):

$$U = 1.00 - 0.0171 \frac{L}{D} \quad \text{(4)}$$

If the ultimate strength of the concrete as developed in test cylinders is designated by f_c, then the ultimate stress of the same concrete in a column will be given by Equation (5), as follows:

$$\frac{P_m}{A} = 1.17\, f_c \left(1.00 - 0.0171 \frac{L}{D}\right) \quad \text{(5)}$$

in which,

P_m = total ultimate load, in pounds, on the column;
A = cross-sectional area of the column, in square inches;
f_c = ultimate strength of the concrete in small test cylinders;
L = length of the column, in inches; and
D = diameter of the column, in inches.

7.—Cast-Iron Columns

The stress-strain diagram for cast iron is of the same general type as that for concrete. Concrete gives warning of impending failure by spalling, the failure itself being of the nature of a combined shear and crumbling. Cast iron gives no notice or warning of impending failure, which is exceedingly violent, almost explosive in character, fragments, sometimes quite large, being violently thrown about.

Like concrete, cast iron is variable in strength. (See Section III.) Uneven cooling, sand holes, blow-holes, internal cold shuts, and unfused chaplet stems, are all likely to occur and also to have an enormous effect in reducing the strength of a cast-iron element, such as a column. By skillful foundry work, allowing proper thickness of metal, and, in general, by good design, cast-iron columns may develop great reliability and strength, and there are being made commercially in the United States columns of such excellence that a comprehensive series of tests on them is in order, as their strength and reliability are far superior to those qualities in the columns that have already been tested and form the basis of the allowable working stresses in use. (See Section IV.)

Frequently, test specimens in the form of cubes are made to determine the strength of the cast iron in columns. For reasons already stated, it is inadvisable to test materials in this form. The American Society for Testing Materials recommends a specimen 1 in. in diameter and from 2.5 to 4 in. long for compression tests of all cast materials. Specimens at least 2 diameters long are excellent for determining the true strength of cast iron, although a small specimen such as this will probably be homogeneous and not have any of the faults of the large castings, and will develop a higher strength than the same metal cast into a column.

Table 2 gives the data on a representative group of cast-iron columns tested by the New York Building Department in 1897.* Although tested so long ago, this group seems to be one of the last, if not the latest series of tests, it being assumed that the tests made during the Nineteenth Century gave results that were final for all time for cast iron. Although data on other tests are available, this small group is sufficiently large for the present purpose, as no detailed analysis of cast-iron columns is intended, the group being used merely as a means of comparison with the various types of concrete columns.

Fig. 3 gives the results of the tests showing the variation of strength with length, and the line representing the equation,

$$\frac{P_m}{A} = 30\,500 - 165\,\frac{L}{D} \qquad (6)$$

* *Engineering News,* January 13th and 20th, 1898.

derived by William H. Burr, M. Am. Soc. C. E., from the combined results of the Watertown tests and another group tested by him at Phœnixville, Pa., in 1898.* Equation (6), for cast-iron columns, which is similar to Equation (2), is obtained by dividing Equation (6) through by 30 500, the result being,

$$U = 1.000 - 0.0055 \frac{L}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

TABLE 2.—CAST-IRON COLUMNS.*

Slenderness ratio, $\frac{L}{D}$	Diameter, in inches.	Thickness of Column Wall, in Inches.			Ultimate Stress, in Pounds per Square Inch.		Ratio: Actual, to theoretical.
		Maximum.	Minimum.	Average.	Actual.	Theoretical†.	
12.7	15	1	1	1	30 830	28 470	1.083
"	15	1 5/16	1	1 1/8	27 700	"	0.973
"	15	1 1/4	1	1 1/8	24 900	"	0.873
"	15 1/8	1 7/32	1	1 1/8	25 200	"	0.885
"	15	1 11/16	1	1 11/16	32 100	"	1.126
"	15	1 1/4	1 1/8	1 3/16	40 400	"	1.420
20.0	7 3/4 to 8 1/4	1 1/4	5/8	1	31 900	27 300	0.832
"	8	1 3/32	1	1 3/64	26 800	"	0.982
"	6 1/16	1 5/32	1 1/8	1 9/64	22 700	"	1.167
"	6 31/32	1 1/8	1 1/16	1 7/64	26 300	"	0.963

* Kent's "Mechanical Engineer's Handbook", 1910 Edition, p. 275.

† From Professor Burr's equation: $\frac{P}{A} = 30\,500 - 160 \frac{L}{D}$.

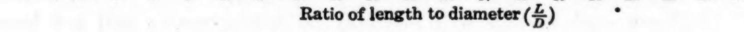

FIG. 3.

8.—STRUCTURAL STEEL COLUMNS

For structural grades of steel, the yield point is 35 000 lb. per sq. in., and the ultimate strength, 55 000 lb. per sq. in., both figures being approximate. In the compression of short specimens, length equal to twice the diameter, a curve the same as the tension curve is produced. However, for any compression member of a commercial structure, the length is sufficiently great to cause failure through buckling of the member at the yield-point stress.

Table 3 and Fig. 4 give the data on a set of ten structural steel columns tested by Professor Talbot.† These columns were duplicates of steel columns that were cast into concrete columns, as reinforcement for the concrete or *vice versa*. The columns were excellently fabricated, tested with the usual preci-

* *Engineering News*, June 30th, 1898.

† Described in *Bulletin No. 56*, Univ. of Illinois Eng. Experiment Station.

sion and thoroughness of Professor Talbot, and, as a group, are exactly fitted for comparison with the other types of columns discussed in this paper. The steel, tested in two 32-in. sections of the angles as used in the columns, showed an ultimate compressive strength of 39 800 lb. per sq. in.

TABLE 3.—STRUCTURAL STEEL COLUMNS.

Length.	Cross-sectional area, in square inches.	Slenderness ratio, $\frac{L}{R}$.	ULTIMATE STRESS, IN POUNDS PER SQUARE INCH. Actual.	Theoretical.*	Ratio : Actual to theoretical column strength.
2 ft. 0 in.	13	6.1	37 450	36 180	1.035
4 ft. 8 in.	"	14.4	33 700	34 800	0.968
"	"	"	34 700	"	0.997
10 ft. 0 in.	"	30.8	31 600	32 080	0.985
"	"	"	32 700	"	1.019
"	"	"	32 600	"	1.016
15 ft. 4 in.	"	47.2	28 300	29 350	0.964
"	"	"	28 900	"	0.981
19 ft. 4 in.	"	59.2	28 800	27 370	1.050
"	"	"	26 500	"	0.968

*From the formula: $\frac{P}{A} = 37\,200 - 512\,\frac{L}{D}$, the equation of the line representative of the ten tested columns. D is the distance back to back of angles, and is approximately the column diameter.

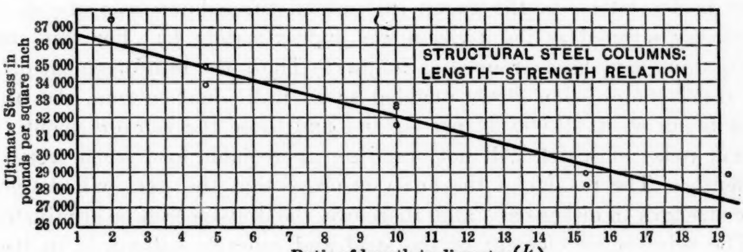

FIG. 4.

The equation for the strength of the columns is:

$$\frac{P_m}{A} = 37\,200 - 167\,\frac{L}{R} \quad \text{.......................(8)}$$

or,

$$\frac{P_m}{A} = 37\,200 - 512\,\frac{L}{D} \quad \text{.......................(9)}$$

in which,

P_m = the ultimate strength of the column, in pounds;
A = the cross-sectional area, in square inches, of the steel in the column;
R = the radius of gyration of the cross-section of the column; and
D = the diameter of the column (back to back of angles).

Dividing through by 37 200, the comparison equation in the form of Equation (2) is obtained:

$$U = 1.000 - 0.0139 \frac{L}{D} \dots\dots\dots\dots\dots\dots\dots (10)$$

Professor Talbot gives the formula:

$$\frac{P_m}{A} = 36\,500 - 155 \frac{L}{R}$$

neglecting the 2-ft. specimen, and, apparently, deriving the equation graphically. Equation (8) is determined by means of the normal equations of the theory of least squares, thus eliminating all personal and other errors present in the graphical method, and gives an incontrovertible result.

9.—Reinforced Concrete Columns

A.—Columns Reinforced with Structural Steel Columns

For simplicity, this type of column will be called the steel and concrete column.

The question as to what percentage of steel reinforcement marks the division between the two groups of columns, one of which may be considered as a concrete column reinforced with steel, and the other a steel column reinforced with concrete, has engaged considerable attention. The allowable working stress is dependent on the group into which the column was placed. (The percentage selected as the division was approximately 4.) For this analysis, the columns have been grouped as follows: Those the reinforcement of which will stand compression stress unaided as an individual column, the ultimate stress being about 35 000 lb. per sq. in., and those the reinforcement (longitudinal rods) of which will not stand an appreciable load when tested independently. The results of the analysis prove that the steel will develop the same strength in either case, and, therefore, the division can be eliminated.

The reinforcing element of the steel and concrete column is in itself an independent structural element, capable of functioning as a column, and can be tested in duplicate to determine its individual strength. The concrete will bear the same load as an unreinforced column of the same cross-section. The determination of the load borne by the reinforced column should be simple, being apparently the sum of the loads borne by the two materials independently. The only possible error would be in using a concrete with an ultimate strain of less than 0.0015, at which strain, structural steel in compression exerts its ultimate or maximum strength. For the longitudinal rod reinforced column this would result in less strength being developed by the reinforced column than the sum of the strengths of the steel and concrete.

The results of tests on twelve columns of the type under discussion have been given by Professor Talbot.* Table 4 gives the data on these columns. The twelve structural steel columns used as reinforcement are from the same series of tests and are duplicate specimens of the steel columns described previously under "Structural Steel Columns". As a result of these tests, Pro-

* *Bulletin No. 56*, Univ. of Illinois Eng. Experiment Station.

fessor Talbot draws the conclusions that the strength of the column is equal to that of the structural steel element plus the strength per unit area of concrete equal to the ultimate strength of the concrete as tested in 8 by 16-in. test cylinders.

TABLE 4.—CONCRETE REINFORCED WITH STRUCTURAL STEEL.

Length, in feet.	ULTIMATE STRESS, IN POUNDS PER SQUARE INCH.					Ratio: Column strength, actual to theoretical.
	Test cylinder.	Average cylinder.	Theoretical steel.*	Theoretical column.†	Actual column.	
4.67	1 350 1 490	1 420	34 800	4 998	4 805 5 010	0.961 1.002
10.00	1 350 1 490	1 420	32 080	4 708	4 236 4 860	0.900 1.032
15.33	1 140 1 260	1 200	29 350	4 210	3 893 4 428	0.925 1.052
19.40	970 1 150	1 060	27 370	3 864	4 076 4 112	1.055 1.064
10.00	2 420 2 520	2 470	32 080	5 640	5 290 5 440	0.938 0.964
10.00	700 660	680	32 080	4 047	4 290 4 407	1.060 1.089

* From the representative equation : $\frac{P}{A} = 37\,200 - 512\frac{L}{D}$.

† See text for derivation of theoretical column strength.

Professor Talbot considers the strength of the steel in the steel and concrete column as the average of the strengths of the two duplicate steel columns of the same length, that were tested. Although the results thus obtained are good and show close agreement between the strength of the steel and concrete column and the sum of the strengths of the concrete and the steel column, better agreement between these two quantities can be obtained. The strength developed by the steel in the steel and concrete column is unknown. A better estimate can be made of its strength by deriving the strength from the entire group of ten steel columns than by assuming it to be equal to the average strength of the one, two, or three steel columns of the same length as the steel and concrete column in question. Therefore, the strength of the steel in the steel and concrete column should be taken as that indicated by Equation (8), the straight line representing the strength of the ten steel columns of different lengths.

To emphasize the exactness with which the strength of the columns coincides with the theoretical strength thus determined, the necessary quantities have been calculated and tabulated in Table 4. The difference between the averages of the theoretical and the actual strengths of the group of reinforced concrete columns is 0.37%, thus proving the validity of the original hypothesis, and showing the care with which the columns were made and tested.

From the length-strength equation of the columns of this type, Equation (11), in the form of Equation (2), is obtained:

$$U = 1.000 - 0.0126\frac{L}{D} \quad \text{.....................(11)}$$

The length-strength relation cannot be obtained directly from the results tabulated in Table 4 as the concrete in the different length columns was of different strengths. Equation (11) is obtained from theoretical column strengths, using one strength of concrete.

B.—Concrete Reinforced with Longitudinal Rods

This type of reinforcement differs from the structural steel type in that the rods, tested alone, cannot support an appreciable load. There is a wide difference of opinion between testing engineers as to the value of this type of reinforcement. Although almost generally accepted as in the average increasing the strength, there is still an opinion that the rods, by their action in the testing machine, of buckling outward and breaking off the concrete covering, hasten column failure. Even those engineers who most favor this type of column, cannot agree on the value of the rods nor can they prove that they have a definite value. When a rodded column shows low strength, it can be attributed to the concrete, or to the rods. The only rational method of settling the question is to draw logical and warranted conclusions from a mathematical analysis of a group of such columns. The following analysis includes all the available test specimens.

With the longitudinal rod reinforced column, there are two variable quantities: The strength due to the concrete and that due to the rods. The strength of the concrete in a column differs from that of similar concrete in a test cylinder, due to the natural variation in the strength in concrete. This variation is so great that for one specimen, or even for several, no conclusion can be reached as to the strength developed by the rods, as the strength of the concrete in individual columns cannot be determined. If the number of columns analyzed is sufficiently large, the variation between the average strength of the concrete in the cylinders and that in the columns can be eliminated. This principle can be illustrated as follows: Suppose two test cylinders are made from each of various batches of concrete of different proportions and strengths. The strengths of the two cylinders from each batch will differ and show no apparent relation. If, say, twenty batches are taken, the average of the two sets of cylinders will come into close agreement. The larger the number of batches, the closer the agreement between the averages of each set of cylinders.

Mathematical Determination of Rod Strength.—The ultimate column stress for a longitudinally steel reinforced column (neglecting length, which will be treated subsequently, all the columns being considered for the present as of the same length), is evidently of the form:

$$\frac{P_m}{A} = S\, r_r + f_o\,(1 - r_r) \qquad \text{(12)}$$

in which,

P_m = the total ultimate load on the column;
A = the cross-sectional area of the column;
r_r = the ratio of steel rod reinforcement;

f_o = the stress in the concrete at the ultimate strength of the column; and,

S = the stress in the steel reinforcement at the ultimate strength of the column.

$(1 - r_r)$ is evidently the ratio of concrete. f_o must be the ultimate strength of the concrete, as in a rodded column, the rods, unsupported, cannot carry any load; that is, the strength of the rods at maximum column strength must be exerted while the concrete is unbroken. The rods, although increasing the strength of the column, cannot increase the strength of the concrete.

Equation (12) can be written in the form :

$$S = \frac{P_m}{A\, r_r} - \frac{f_o}{r_r}(1 - r_r) \qquad (13)$$

From the results of column tests, all the values of the variables in Equation (13) are obtained with the exception of S, which, therefore, is determined, being a one valued explicit function of the other variables. The value of S determined from any individual column is valueless, as it depends on the actual strength of the concrete in the column, which is not known.

However, by increasing the number of test specimens, the average of the ultimate strengths of the concrete in the test cylinders approaches the average strength of the concrete in the column. Consequently, the average of the values of S obtained from a number of individual columns will approach the strength of the reinforcing rods at the ultimate strength of the column.

Equation (13) can be written as follows:

$$S = \frac{P_m}{A\, r_r} - f'_o \left(\frac{1}{r_r} - 1\right) \qquad (14)$$

For N columns, there are N equations, the summation of equations of the form of Equation (14), being:

$$S = \frac{1}{N} \sum_N \frac{P_m}{A\, r_r} + \frac{1}{N} \sum_N f'_o \left(1 - \frac{1}{r_r}\right) \qquad (15)$$

Now,

$$f'_{om} = f'_{cm} + x$$

in which,

f'_{cm} = the ultimate stress of the concrete of Cylinder M;

f'_{om} = the ultimate stress of the concrete of Column M;

x = therefore, the difference between the ultimate stress of the concrete in the columns and the ultimate stress of the concrete in Cylinder M.

Equation (15) may then be written as follows:

$$S = \frac{1}{N} \sum_N \frac{P_m}{A\, r_r} + \frac{1}{N} \sum_N \left(f'_{cm} + x\right) \left[1 - \frac{1}{r_r}\right] \qquad (16)$$

therefore,

$$S = \frac{1}{N} \sum_N \frac{P_m}{A\, r_r} + \frac{1}{N} \sum_N f'_{cm} \left(1 - \frac{1}{r_r}\right) + \frac{1}{N} \sum_N x \left(1 - \frac{1}{r_r}\right) \qquad (17)$$

As already stated, the average ultimate strength of the concrete cylinder is equal to the average ultimate stress of the concrete column, or these two averages approach each other. Therefore, $\sum_N x$ approaches zero, and can be neglected in a set of columns as large as that included in this analysis. In consequence, $\frac{1}{N} \sum_N x \left(1 - \frac{1}{r_r}\right)$ is very small or practically zero. Therefore:

$$S = \frac{1}{N} \sum_N \frac{P_m}{A\, r_r} + \frac{1}{N} \sum_N f'_{cm} \left(1 - \frac{1}{r_r}\right) \dots\dots\dots\dots (18)$$

From Equation (18), it may be noted that the stress in the steel at ultimate column stress can be determined closely from the following quantities:

P_m = the ultimate strength of the column;
A = the cross-sectional area of the column;
r_r = the ratio of rod reinforcement; and
f'_c = ultimate stress of the concrete of the same batch as the column concrete moulded into a cylinder.

Table 5 gives the data on the group of columns analyzed. Applying Equation (18) to this group of columns, the value of S obtained is 31 000 lb. per sq. in., which means that the average stress in the steel reinforcing rods at ultimate column stress was likewise 31 000 lb. per sq. in.

TABLE 5.—LONGITUDINALLY REINFORCED CONCRETE COLUMNS.

Source.	Column diameter, in inches.	Ratio, column length to diameter, $\frac{L}{D}$.	Percentage of longitudinal reinforcement (rods).	Ultimate Stress, in Pounds per Square Inch. Concrete strength, average of cylinders.	Column strength.	Apparent stress on rods at ultimate column strength.	Theoretical column strength.	Ratio: Actual to theoretical column strength.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A. C. I.*	20.2	7.2	1.00	3 206	3 300	12 000	3 490	0.945
			0.98		4 200	105 000	3 480	1.207
			2.04	3 256	3 620	21 300	3 819	0.948
			2.02		3 760	28 500	3 812	0.986
			4.08	2 887	4 565	44 000	4 035	1.132
			4.07		4 500	42 500	4 030	1.117
T.†	9 by 9 (square)	14.5	1.55	1 150	1 280	9 550	1 597	0.801
		10.6	1.48	2 004	2 335	24 400	2 432	0.960
		10.6	1.49	1 368	1 607	17 450	1 839	0.874
W_{800}‡	10.0	9.8	2.35	1 960§	2 310	17 000	2 638	0.875
			2.35		2 310	17 000	2 638	0.875
			2.35		2 695	33 400	2 638	1.022

* Am. Concrete Inst., *Proceedings*, February, 1915.

† Univ. of Illinois, *Bulletin 10*, Eng. Series.

‡ Univ. of Wisconsin, *Bulletin 300*, Eng. Series.

§ Equivalent cylinder strength, equal to strength of concrete in columns. Test cylinders are the best available indication of the strength of the concrete in the individual column. When it is evident that the cylinder strengths in any series of tests for whatsoever reason are not equal to the strength of the concrete in the column, then this latter strength may be determined in some better way than being equal to test cylinder strength. The ratio of the cylinder strength and unreinforced concrete column strengths in the W_{800} series of tests are at variance with the ratios obtained in all other series, including the supplementary W_{466} group. In all other groups of tests, the column and cylinder strengths are very nearly equal; in the W_{466} group the average column strength is 7% higher than the cylinder strength. In the W_{800} series, the average column strength is only 0.874 of the average cylinder strength. The value given in Table 5 is, therefore, taken as 0.874 of the cylinder strength.

Using the value of 31 000 lb. per sq. in. for the rods, and the ultimate stress for the concrete in the cylinders as that of the concrete in the columns, the percentage of the theoretical to the actual column strength was calculated, with the results shown in Column (9) of Table 5. These values were plotted with the values of $\frac{L}{D}$ for the columns, the line representative of these points giving the reduction in strength with length for this type of column. Assuming that the value of the stress in the rods, at ultimate column stress, varies as the strength of the column as a whole, the rods have a strength of 39 700 lb. per sq. in. for a column of infinitely small length. This value agrees well with the ultimate compressive stress of short sections of the rods tested independently and is conclusive evidence that the rods develop their full ultimate compressive stress (yield-point stress) in columns of this type, corrected for variation in the strength of the column with length.

The length-strength equation for the longitudinally reinforced column, in the form of Equation (1), is:

$$U_0 = 1.28 - 0.034 \frac{L}{D} \qquad \text{......................(19)}$$

Equation (19) reduces to:

$$U = 1.00 - 0.0265 \frac{L}{D} \qquad \text{......................(20)}$$

It can be seen by comparing Equation (20) with the similar equations for the other types of columns that the reduction in strength with length for this type of column is rapid.

C.—Concrete Reinforced with Spirals*

Load applied to concrete reinforced with spirals produces secondary stresses in the spiral. A longitudinal compression in the concrete produces a tension in the spiral. The increase in ultimate strength of concrete thus reinforced is due to the reaction of the spirals to the tension stresses produced in them. Due to the indirect or inverse manner in which the spiral adds strength to the column, engineers were skeptical about accepting spiral reinforcement. Numerous tests, however, have demonstrated that spirals do add great strength to concrete; and spiral reinforcement, if used in conjunction with longitudinal rod reinforcement, is generally a recognized method of reinforcement. Many engineers have yet to be converted to the use of spiral reinforcement, and especially to the value of longitudinal rods in spirally reinforced columns. This is brought out by the 1917 Specifications, which designate the necessary percentage of rods as from 1 to 5, without allowing a higher working stress in a column with the 5% of rods than in one with the 1% of rods.

The concrete outside the spiral, necessary for fire protection, begins to scale and becomes useless for taking load at the ultimate strain of the concrete (about 0.0015 in. per in.). As the ultimate strain of spirally rein-

* See Section II for a complete discussion of the internal behavior of spirally reinforced concrete under load.

forced concrete is much more than that of unreinforced concrete, the shell contributes no strength to the column at ultimate load. Consequently, in the following analysis, the column will be considered as having no concrete outside the spiral.

No laws showing the relation between the increase of the strength of concrete due to spiral reinforcement and the percentage of spiral, ultimate strength of spiral, etc., have been propounded. All investigators have assumed that spiral reinforcement adds a certain definite unit strength to that of the concrete, irrespective of the strength of the concrete. (Professor Withey deduces an exception to this from the fact that concrete having a strength of 4 900 lb. per sq. in. or more will not have its strength noticeably increased by spiral reinforcement). It is also generally understood that high carbon steel wire as spiral reinforcement gives more strength to the concrete in a column than wire of low carbon or structural grade steel.

That spiral reinforcement should add a certain unit strength does not seem the simplest nor the most logical assumption in attacking the problem. It appears more logical to assume that the spiral increases the strength of the concrete in the column by a certain function of the strength of the unreinforced concrete. A constant multiplier is the simplest function to assume; that is, that the increase in strength of the concrete for a fixed percentage of spiral is a definite percentage of the strength of the concrete.

This is expressed by Equation (21):

$$f_{os} = (1 + k\, r_s)\, f_c \quad \text{.......................} \quad (21)$$

in which,

f_{os} = ultimate strength of the spirally reinforced concrete;
f_c = ultimate strength of the unreinforced concrete;
r_s = ratio of spiral reinforcement; and
k = a constant.

Proceeding on this assumption, the percentage increase in strength of the concrete in the column over the strength of the concrete in corresponding test cylinders has been calculated for the group of columns given in Table 6. The percentage increase has been plotted with the percentage of spiral reinforcement in Fig. 5. A representative straight line has been drawn for the points and it may be noted that all the points with the exception of the two *M* groups, 1, 2, and 3 and 14, 15, and 16, apparently are in agreement with the line. It must be remembered that the strength of the columns will vary greatly due to the variation in strength of the concrete and that, therefore, the points representing individual columns will not coincide with the representative line. The agreement of individual points and the representative line cannot be closer than the strength agreement of individual concrete test specimen strengths with the average strength.

The specimens (W_{466} = 27, and W_{466} = 28) were made from concrete having an ultimate strength of about 4 900 lb. per sq. in. As there were other columns made by the same investigator in the same group, of concrete of lower strength, that had their strength increased according to the line of Fig. 5, it may be concluded that concrete of such high strength does not

gain appreciably in strength due to spiral reinforcement. The specimens made of concrete having a strength of about 4 000 lb. per sq. in., show a slightly lower strength than that indicated from the curve, allowing for possible variation in strength of concrete. At some point, possibly between 4 000 and 4 900 lb. per sq. in., the additional strength due to spiral reinforcement ceases to be proportionately as large as that of concrete of lower

TABLE 6.—SPIRAL REINFORCED CONCRETE COLUMNS.

No.	Source.	Column dimensions (of core): diameter × length, in inches.	Ratio: length to diameter, $\frac{L}{D}$	Spiral Reinforcement. Amount, percentage.¶	Spiral Reinforcement. Yield-point strength, in pounds per square inch.	Spiral Reinforcement. Ultimate strength, in pounds per square inch.	Test Cylinder, Ultimate Strength, in Pounds per Square Inch. Individual.	Test Cylinder, Ultimate Strength, in Pounds per Square Inch. Average.	Columns, ultimate strength, in pounds per square inch.	Ratio: Column to cylinder strength.	Ratio: Actual column to theoretical column strength.
(1)	M *	14 by 120	8.6	0.46	70 000	137 000	2 785	1 965	3 321	1.69	1.34
(2)							1 273		3 280	1.67	1.32
(3)							1 838		3 415	1.85	1.47
(4)	W_{466}†	10 by 102	10.2	0.50	81 000	110 000	2 040	1 750	2 330	1.32	1.05
(5)							1 460		2 140	1.22	0.96
(6)	T ‡	12 by 120	10.0	0.85	60 000	90 000	1 600	1 650	2 503	1.56	1.05
(7)							1 700		2 506	1.47	0.99
(8)	T ‡	12 by 120	10.0	0.845	38 000	53 000	1 350	1 433	2 080	1.54	1.04
(9)							1 500		2 203	1.47	0.99
(10)							1 450		2 220	1.53	1.03
(11)	A §	20 by 144	7.2	0.94	61 000	83 000	2 868	2 945	4 760	1.62	1.05
(12)							3 083		4 570	1.55	1.01
(13)							2 885		5 210	1.77	1.15
(14)	M *	14 by 60	4.3	0.96	67 000	134 000	....	2 374	4 900	2.06	1.33
(15)							2 569		4 973	2.10	1.35
(16)							2 178		4 595	1.96	1.26
(17)	M *	14 by 120	8.6	0.96	67 000	134 000	2 693	3 103	4 608	1.48	0.96
(18)							2 113		4 593	1.48	0.96
(19)							4 504		5 049	1.63	1.05
(20)	M *	14 by 240	17.2	0.96	67 000	134 000	....	2 165	2 939	1.36	0.88
(21)							1 529		3 602	1.66	1.07
(22)							2 800		4 266	1.97	1.27
(23)	W_{466}†	10 by 102	10.2	1.00	96 000	133 000	1 780	1 770	2 680	1.51	0.96
(24)							1 760		2 600	1.47	0.94
(25)	W_{466}†	10 by 102	10.2	1.00	96 000	133 000	4 060	4 070	5 950	1.47	0.94
(26)							4 080		5 760	1.41	0.90
(27)	W_{466}†	10 by 102	10.2	1.00	96 000	133 000	4 830	4 880	5 760	1.19	††
(28)							4 930		4 920	1.00	
(29)	T ‡	12 by 120	10.0	1.64	54 000	73 000	1 150	1 150	2 068	1.81	0.935
(30)	T ‡	12 by 120	10.0	1.67	115 000	152 000	1 900	1 900	3 800	2.00	1.02
(31)	T ‡	12 by 120	10.0	1.68	54 000	73 000	1 900	1 900	3 793	1.99	1.015
(32)	T ‡	12 by 120	10.0	1.71	115 000	150 000	1 800	1 800	3 404	1.89	0.953
(33)	T ‡	12 by 120	10.0	1.73	115 000	150 000	1 400	1 400	2 718	1.94	0.976
(34)	M *	14 by 120	8.6	1.95	68 000	95 000	3 510 2 318 1 850	2 559	5 256	2.06	0.97
(35)									5 408	2.11	0.995
(36)	W_{300}‖	10 by 120	12.0	2.00	98 000	130 000	1 845** 2 160**	2 003	4 660	2.33	1.10
(37)	W_{300}‖	10 by 120	12.0	2.00	98 000	130 000	1 685** 2 160**	1 923	4 390	2.28	1.06
(38)	W_{300}‖	10 by 120	12.0	2.00	98 000	130 000	1 765** 2 070**	1 918	3 660	1.91	0.89
(39)	W_{300}‖	10 by 120	12.0	2.00	98 000	130 000	2 050** 1 720**	1 885	3 410	1.81	0.845

* *Proceedings*, Am. Concrete Inst., 1916, p. 200.

† *Bulletin No. 466*, Eng. Series, Univ. of Wisconsin.

‡ *Bulletin No. 20*, Eng. Series, Univ. of Illinois.

§ *Proceedings*, Am. Concrete Inst., February, 1915.

‖ *Bulletin No. 300*, Eng. Series, Univ. of Wisconsin.

¶ Based on volume of concrete within spiral.

** Equivalent cylinder strength. (See Table 5.)

†† Omitted from computations. Spiral does not give proportionate increase of strength to this high strength concrete.

strength. As there is only two reinforced columns of concrete of high strength on which to base conclusions, it cannot be decided whether the columns did not show an increased strength due to the high strength of the concrete or to some other property which might be independent of its high strength.

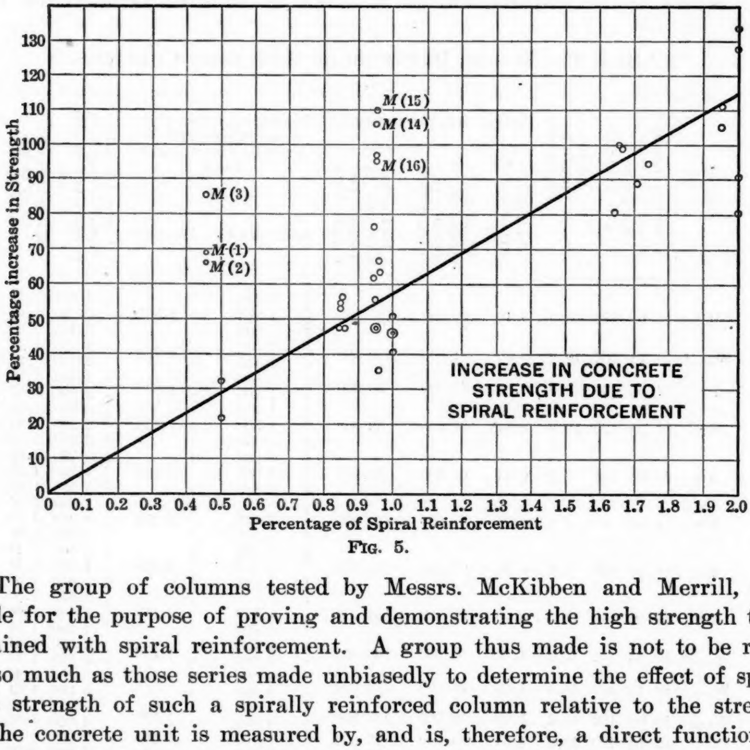

FIG. 5.

The group of columns tested by Messrs. McKibben and Merrill, were made for the purpose of proving and demonstrating the high strength to be obtained with spiral reinforcement. A group thus made is not to be relied on so much as those series made unbiasedly to determine the effect of spiral. The strength of such a spirally reinforced column relative to the strength of the concrete unit is measured by, and is, therefore, a direct function of, the strength of the test cylinders. If by any means the apparent strength of the test cylinders is lowered, then, as a result, the relative and apparent strength of the column is increased, and, therefore, the increase in strength of the concrete in the column due to the spiral, is increased. Several cylinders in this series that failed at high values were discarded by the authors as being abnormal. The practice of arbitrarily discarding discordant results cannot be too severely condemned, as the only legitimate reason for such practice is the proof of that abnormality. If a cylinder tested low, and it could be seen that some defect had caused the reduction in strength, then it would be permissible and proper to exclude the cylinder from all computations. To exclude a cylinder because of high strength, however, is preposterous, as positively no defect could be present. In this analysis, all the test cylinders, including those discarded, are included in the averages in determining the strength of the concrete, resulting in two cases out of the five in bringing the columns (that is, groups of three columns) from marked disagreement into good agreement with the representative line. From this

series of tests, five groups of columns are available for comparison in Fig. 5. Three groups agree with the representative line and the other two groups disagree, showing a much higher strength than the straight line would indicate. A minority of the specimens in a test such as the one under consideration, made for the purpose of obtaining as high a strength as possible, showing an abnormally high and non-concomitant strength, should have only a small influence on the results to be drawn from the several series of columns as a whole. It has been considered that the best procedure would be to omit from the present calculations the two groups (M-1, 2, 3 and M-14, 15, 16), which show such abnormally high strengths.

Even if these two groups of columns of high strength can be shown, in the future, to have possessed this high strength due to some property of the concrete, the spirals, or the method of treatment of the concrete, etc., until this strength can be shown capable of reproduction at will, the reason for the high strength must be considered as accidental and to have no bearing in the derivation of allowable working stresses.

Equation (21) is that of the representative straight line, giving the increase in the strength of concrete due to spiral reinforcement, as follows:

$$f_{os} = (1 + k\, r_s)\, f_c$$

Transposing, etc., there is obtained:

$$k = \frac{1}{r_s}\left(\frac{f_{os}}{f_c} - 1\right) \qquad (22)$$

The term, $\frac{f_{os}}{f_c} - 1$, is 0.01 of the percentage increase of the concrete due to spiral reinforcement. The value of k was calculated for each column, the mean value being 0.573 ± 0.016 and Equation (21) then becomes:

$$f_{os} = (1 + 0.573\, r_s)\, f_c \qquad (23)$$

It has been generally understood that the increase in the strength of concrete due to spiral reinforcement is a function of the ultimate strength of the wire constituting the spiral and that wire of high ultimate strength (high-carbon steel) will give superior column strength. This was a natural error to be expected in an initial abstract consideration of the problem. Further, a coincidence in an important group of tests apparently bore out the supposition, although not in as marked a degree as was expected.

Examining all the columns included in Table 6, it may be noted that although the yield-point strength and the ultimate strength of the spirals varied greatly, there is no relation between the variation in strength of these quantities and the increase in strength due to the spiral reinforcement. Even if such a relation existed, it would be difficult to discern, owing to the introduction of the strength variation inherent in concrete.

The group of seven columns tested by Professor Talbot affords an excellent proof of the independence of the increase in column strength and the ultimate strength or yield-point strength of the spiral, and are sufficient, without the consideration of the whole group of spirally reinforced columns, to prove this independence. In analyzing the results of these tests, Professor Talbot assumed that the spiral gives to the column a constant increase in strength

per unit of column area, independent of the strength of the concrete, and it so happened that, in all but three instances, the concrete of low strength was reinforced with a spiral of low strength, and the concrete of high strength was reinforced with a spiral of high strength. The apparent result is to give a greater increase in strength with the spiral of higher strength. In Columns 31 and 32, however, of which the concrete is of the same strength, the spiral with the low ultimate strength caused a higher column strength than that of high ultimate strength. The average value of k from Equation (22) for the columns reinforced with the high-carbon wire differs by less than 1% from its average value for the columns reinforced with the low-carbon wire. The range between the strength of the low-carbon and the high-carbon spiral for this group of seven columns is the extreme for the spirals in all the columns included in Table 6. The constant, k, being a measure of the increase in strength due to spiral reinforcement, this increase is independent of the strength of the spirals for the extreme limits of 60 000 to 150 000 lb. per sq. in. for the ultimate strength and a concomitant variation of from 38 000 to 115 000 lb. per sq. in. in the yield-point stress.

There is, therefore, waste of strength in using high-carbon reinforcement; low steel is preferable as it does the same work, is easier to manufacture, and is not as easily harmed by accidental deformation.

The method of deriving the length-strength equation from the spiral column was similar to that used for the columns reinforced with longitudinal rods; that is, the ratio of theoretical (according to Equation (23)) to actual column strength, expressed in percentage, was determined for each column. The percentages are then plotted with the length of the column, resulting in Equations (24) and (25):

$$U_0 = 1.27 - 0.022 \frac{L}{D} \qquad (24)$$

$$U = 1.00 - 0.0173 \frac{L}{D} \qquad (25)$$

The general equation of the spirally reinforced column then may be written:

$$\frac{P_m}{A} = 1.27 \, (1.00 + 0.573 \, r_s) \, f'_c \left(1.000 - 0.0173 \frac{L}{D}\right) \qquad (26)$$

in which,

P_m = the total ultimate column load, in pounds;
A = the cross-section of the column, in square inches;
r_s = the ratio of spiral reinforcement;
f'_c = the ultimate strength of the concrete, as determined in test cylinders;
L = the length of the column, in inches (or feet); and
D = the diameter of the spiral, in inches (or feet).

Equation (26) is an empirical equation derived as an average for the spirally reinforced column from a set of points. From any set of values, however, even if they are independent of any law of formation, an average

value can be found. Although the points for the individual columns, in a general way, agree with the representative line, they vary to such a large extent from it that the variation could be the result of a fundamental disagreement with the line or from the inherent variation of strength of concrete. The proof that this is the correct equation is neatly applied by the method of measuring the variation in the strength of materials, as developed and described in Section III. The individual variation in strength of all the columns in Table 6 from that given by Equation (23) has been obtained. The measure of this variation is almost the same as it would be if the columns were made of concrete without any reinforcement, showing the variation in the strength of the columns to be due to the inherent variation in the strength of the concrete and proving Equation (23) to be correct.

D.—Columns Reinforced with Spirals and Longitudinal Rods

Authorities differ as to the value of longitudinal reinforcing rods in spirally reinforced columns. It is a common impression that the rods buckle, bending outward, thus bearing on the spiral and causing it to fail. That the rods generally caused an increase in strength was admitted and could be proved from the results of tests. This is a simple process, as the average strengths of two groups of columns that were identical, except that one set had no reinforcement, can be compared, with the result that the group with rods is always the stronger. The contention, however, is and has been, that, although the average is stronger, in an individual case the rods may add no strength, or, worse, decrease it. The inherent variation in the strength of concrete made the problem complex and it is impossible to measure the strength that the rods developed in any one column. The statement that a low column strength was the result of the rods could be disputed, but not disproved. The method of determining the increase in the strength of the concrete due to spiral reinforcement being at fault added to the confusion.

No method of eliminating the effect of the variation in strength of concrete has ever before been used. The column was considered a unit, the strength of the column as a unit being considered somewhat as a function of the number of rods used as reinforcement. The actual stress developed by the rods, considered as independent units, however, was never attempted.

The strength of the column is a function of the following: The strength of the concrete, the strength added to the concrete by the spiral reinforcement, and the stress in the longitudinal rods at ultimate column strength. In each column, therefore, there are three unknowns. It is known, however, that the spiral increases the strength of the concrete in a definite manner as given by Equation (23):

$$f_{os} = (1 + 0.573\, r_s)\, f_c$$

Equation (23) is valid when rods are used, as they cannot increase the strength of the concrete.

There are now, therefore, only two unknowns, the strength of the concrete in the column and the stress in the rods at ultimate column strength. As in the rod reinforced column (previous to the development of the formula for the

strength of the rods), the strength of the concrete in an individual column is unknown. Its strength will not be the same as that of the concrete in the test cylinders from the same batch of concrete, and to determine the strength of the rods by the difference between the strength of the column and that due to the spirally reinforced concrete, assuming that the column concrete is of the same strength as the cylinder concrete, leads to absurd results that are sometimes negative. The unit strength of the rods at ultimate column strength thus calculated is given in Column (9) of Table 7. It will be noticed that these quantities are not the true stress in the rods, as that is impossible to determine, and, on account of the large variation in these figures, it is not to be concluded that the strength developed by the rods is such a variable quantity. It will be shown subsequently that the rods develop a very constant strength.

The average of the values given in Column (9) of Table 7 gives the true average value that the rods develop at ultimate column strength, as, in the averaging, the inherent variation in strength due to the concrete is eliminated. This can be understood by considering the demonstration given for columns reinforced with rods. The average of the values in Column (9) of Table 7, is 36 500 lb. per sq. in., which figure gives the impression that it is correct, and that, therefore, the whole basis on which the result was derived was also correct.

Obtaining the length-strength relation by the same method used for the spiral reinforced column, the equation:

$$U_0 = 1.20 - 0.022 \frac{L}{D} \qquad (27)$$

and the reduced equation:

$$U = 1.000 - 0.0183 \frac{L}{D} \qquad (28)$$

are obtained.

Assuming that the strength of the rods varies with that of the column as a whole, the value for the rods for a column of infinitesimal length is 43 800 lb. per sq. in. The exact figure for the strength of the rods cannot be obtained, because of the method that must be used. The value, 43 800 lb. per sq. in., however, is proof that the rods exert their yield-point stress at the ultimate column stress, and, as a corollary, it may be stated that, from these results, it is conclusive that the rods do not cause any reduction in the strength of the column or of its components.

The general equation to express the strength of the concrete column reinforced with spiral and longitudinal rods is, as follows:

$$\frac{P_m}{A} = 1.20 \left[(1 - r_r)(1 + 0.573\, r_s)\, f'_c + 36\,500\, r_r\right]\left[1.00 - 0.0183 \frac{L}{D}\right] \qquad (29)$$

in which,

r_r = the ratio of longitudinal steel rod reinforcement;
r_s = the ratio of spiral reinforcement;
P_m = the total load borne by the column at ultimate strength;
A = the cross-sectional area of the column within the spiral;

L = the length of the column;
D = the diameter of the column within the spiral; and
f'_c = the ultimate test cylinder strength.

TABLE 7.—ROD AND SPIRAL REINFORCED CONCRETE COLUMNS.

Source.	Column dimensions (of core): diameter × length, in inches.	Ratio: Length to diameter, $\frac{L}{D}$	Reinforcement rods§, percentage.	Reinforcement spirals§, percentage.	TEST CYLINDER: ULTIMATE STRENGTH, IN POUNDS PER SQUARE INCH.		Columns: Ultimate strength, in pounds per square inch.	Stress on rods at ultimate column strength, in pounds per square inch.¶	Ratio of actual to theoretical column strength.
					Individual.	Average.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
A *	20 by 144	7.2	0.98	0.94	3 396 2 773 2 860	3 009	4 710 4 910 5 140	24 500 43 900 68 400	0.914 0.960 1.000
A *	20 by 144	7.2	0.99	1.88	3 035 2 883	2 959	6 025 6 495	—500 46 900	0.913 0.970
A *	20 by 144	7.2	1.98	0.94	3 185 2 575 2 675	2 812	5 050 4 785 5 885	37 600 24 200 79 800	0.975 0.922 1.144
W †$_{466}$	10 by 102	10.2	2.00	0.50	2 100 1 400	1 760	3 320 3 280	55 000 53 000	1.150 1.137
W †$_{466}$	10 by 102	10.2	2.00	1.00	2 100 1 890	1 995	4 050 3 760	48 500 34 000	1.088 1.012
W ‡$_{300}$	10 by 120	12.0	3.50	2.00	1 825‖ 1 945‖	1 885	4 470	16 300	0.929
W ‡$_{300}$	10 by 120	12.0	3.50	2.00	1 790‖ 2 170‖	1 980	4 200	2 140	0.843
W ‡$_{300}$	10 by 120	12.0	3.50	2.00	2 000‖ 1 975‖	1 988	4 970	24 000	0.985
W ‡$_{300}$	10 by 120	12.0	3.50	2.00	2 160‖	2 160	5 360	25 400	1.000
W †$_{466}$	10 by 102	10.2	3.77	0.50	2 240 2 120	2 180	4 240 4 080	30 900	1.065 1.025
W †$_{466}$	10 by 102	10.2	3.77	1.00	1 880 1 720	1 800	4 050 4 340	31 150	1.012 1.085
A *	20 by 144	7.2	6.15	0.95	3 095 2 670 2 775	2 850	6 180 5 750 6 350	33 500 26 500 36 300	0.930 0.861 0.955
W †$_{466}$	10 by 102	10.2	5.83	1.00	2 020 1 520	1 770	5 760 4 980	53 700 40 500	1.238 1.073
W †$_{466}$	10 by 102	10.2	6.11	0.50	2 110 2 000	2 055	5 190 5 050	44 500 42 400	1.128 1.098
W †$_{466}$	10 by 102	10.2	6.11	1.00	1 660 1 710	1 685	4 790 4 580	37 800 34 400	1.067 1.000
W †$_{466}$	10 by 102	10.2	6.11	1.96	2 270 2 690	2 480	6 510 6 650	25 100 27 600	0.928 0.947
W †$_{466}$	10 by 102	10.2	6.11	1.00	4 720 4 080	4 400	7 480 7 100	....	....
W †$_{466}$	10 by 102	10.2	6.11	1.00	4 900 4 920	4 910	8 050 8 250	....	....
W †$_{466}$	10 by 102	10.2	8.00	1.00	2 380 2 350	2 365	6 760 7 090	41 700 45 900	1.091 1.143
W †$_{466}$	10 by 102	10.2	8.00	1.96	2 310 2 470	2 380	7 250 6 680	30 800 23 700	0.966 0.892
W †$_{466}$	10 by 102	10.2	10.12	1.96	2 280 2 330	2 305	6 190 7 990	17 350 35 100	0.786 0.847

* *Proceedings*, Am. Concrete Inst., February, 1915.
† *Bulletin No. 466*, Eng. Series, Univ. of Wisconsin.
‡ *Bulletin No. 300*, Eng. Series, Univ. of Wisconsin.
§ Based on volume of concrete within spiral.
‖ Equivalent cylinder strength. (See Table 5.)
¶ Not corrected for column length. Average for the total column of values = 36 500 lb. per sq. in.

E.—Columns Reinforced with Spiral and Cast Iron

This general type of column was first suggested and used on the Continent by Dr. von Emperger of Austria.*

Spirally reinforced concrete undergoes considerably more strain at ultimate load than plain concrete, which has an ultimate strain ranging from 0.0006 to 0.0016 in. per in., whereas spirally reinforced concrete has an ultimate strain of 0.0035 to 0.0150 in. per in. (See Section II.) The stress in cast iron of ordinary grade at a strain of 0.0016 is only 16 000 lb. per sq. in., whereas for a strain of 0.0035, its stress is about 35 000 lb. per sq. in., and may be as much as 90 000 lb. per sq. in. Morley† gives a stress-strain curve for such a specimen. It is evident, therefore, that a longitudinal structural element of cast iron in a spirally reinforced column would exhibit at least the strength of structural steel, and considerably more than it would in a plain concrete column. With the better grade of castings that are being introduced in the United States, there is reason to expect a new era for the use of cast iron.

TABLE 8.—CONCRETE COLUMNS REINFORCED WITH SPIRAL AND CAST IRON.

Length of column, in feet.	Cast-Iron Reinforcement.		Percentage of Areas of Net Column Area Within Spiral.				Column Load, in Pounds.		Ultimate Stress, in Pounds per Square Inch.	
	Diameter outside, in inches.	Thickness, in inches.	Spiral.	Rods.	Cast iron.	Concrete.	At first sign of failure.	Maximum (ultimate).	Column (mean).	Cylinder.
6	6	¾	0.71	0.68	12.7	86.6	436 500	1 057 000	10 880	4 380
6	6	¾	0.71	0.68	12.7	86.6	776 000	1 026 000	10 560	4 380
8	6	¾	0.71	0.68	12.7	86.6	436 500	940 000	9 680	3 720
8	6	¾	0.61	0.58	10.9	88.5	570 000	1 031 500	9 110	3 720
8	5	1	0.65	0.62	11.9	87.0	630 500	925 000	8 730	4 150
8	7	⅝	0.79	0.76	14.4	84.9	565 500	920 000	10 560	4 150
10	6	¾	0.71	0.68	12.7	86.6	643 500	911 000	9 380	3 750
10	6	¾	0.71	0.68	12.7	86.6	625 000	940 000	9 680	3 750
10	6	¾	0.71	1.26	12.7	86.2	776 000	896 000	9 230	4 300
10	6	¾	0.71	1.26	12.7	86.2	727 500	1 066 000	10 980	4 300
10	(*)	(*)	0.61	1.08	11.8	87.0	850 000	1 071 800	9 480	4 300
10	(*)	(*)	0.61	1.08	11.8	87.0	576 000	996 000	8 860	4 300
12	6	¾	0.71	0.68	12.7	86.6	732 500	951 500	9 800	4 250
12	6	¾	0.71	0.68	12.7	86.6	630 500	909 000	9 360	3 900
14	6	¾	0.71	0.68	12.7	86.6	680 500	888 500	9 150	4 250
14	6	¾	0.71	0.68	12.7	86.6	679 000	837 500	8 520	4 400
8	6	¾	....	....	100.0	....	480 000	480 000	39 000	......
8	6	¾	....	....	100.0	....	540 000	540 000	43 800	......

*Special I-section.

Table 8 and Fig. 6 give the data on a group of sixteen columns of the von Emperger type. These columns were tested at the Pittsburgh Branch of the U. S. Bureau of Standards for L. J. Mensch, M. Am. Soc. C. E., and have been described by him.*

The cast-iron reinforcement was of two types, one being a hollow tube or cylinder, the other having an I-section. The hollow-core type is advan-

* *Beton und Eisen*, 1912, p. 118.

† "Theory of Structures."

* "Tests of Concrete Columns with Cast-Iron Reinforcement," *Proceedings*, Am. Concrete Inst., 1917, p. 22.

tageous in that it gives a larger radius of gyration and consequently greater stiffness and strength for the same cross-section. The hollow core can be used, if sufficiently large, for piping or ventilation. The maximum load is carried by the column at the time the concrete fails, and this occurs before the spirals fail in tension. (See Section II.) If compression of the column is continued, the spiral will begin to fail in tension. Further compression causes the failure of the cast iron inside, which is indicated by a pronounced thud. In consequence of this method of failure, the concrete, at maximum column load, is carrying the same stress as it would have carried if the cast iron had not been used as reinforcement, which is that of the concrete as increased by the spiral reinforcement. The added strength of the von Emperger column is due to the cast-iron core, and this may be found by difference. Mr. Mensch follows this method, using as the strength of the concrete the average ultimate stress of the two concrete columns reinforced with spiral only (and the same percentage of rods, as the remainder of the von Emperger type tested). The concrete in these two columns, however, was much stronger than that used in the von Emperger type of columns, as shown by the strengths of the test cylinders. It would be much better to

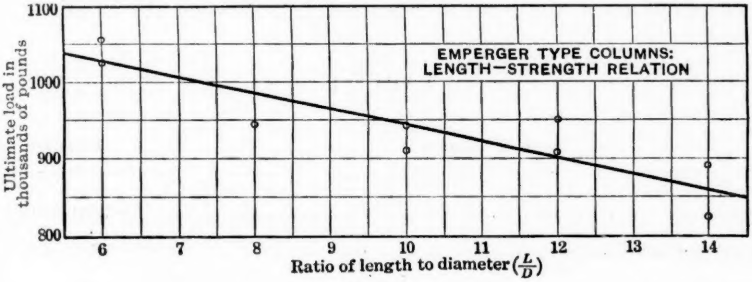

FIG. 6.

use the strength of the concrete in the test cylinders, increased in strength by the spiral, as given by Equation (23). The net result of the latter treatment is to show a smaller stress in the cast iron when the column is bearing maximum load, the value being 37 400 lb. per sq. in., computed by the latter method. This value gives an accurate means of obtaining the strain at which reinforced concrete fails. From the stress-strain diagram of the cast iron similar to that used as reinforcement in the columns mentioned, it is seen that the strain corresponding to the stress of 37 400 lb. per sq. in. is 0.0034 in. per in.

Plotting the strengths of the nine columns of the same cross-section, the length-strength relation obtained is:

$$U_0 = 1.220 - 0.0219 \frac{L}{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

or reduced,

$$U = 1.000 - 0.0179 \frac{L}{D} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

The equation for the column of this type can be expressed as:

$$\frac{P_m}{A} = \left(1.220 - 0.0219\,\frac{L}{D}\right)\left[f_{os}\,(1 - r_i - r_r) + I r_i + w r_r\right] \ldots\ldots (32)$$

in which,

P_m = the total ultimate load on the column, in pounds;
A = the total cross-sectional area of the column (not including the space in the core), in square inches;
L = the length of the column, in inches or feet;
D = the diameter of the column, in inches or feet, within the spiral;
r_i = the ratio of cast-iron reinforcement;
r_r = the ratio of longitudinal steel reinforcement;
w = the yield point of the steel reinforcement (this may be assumed to be 35 000 lb. per sq. in.); and
I = the stress in the cast iron at the strain of 0.0034.

f_{os} is given by Equation (23).

F.—Comparison of the Various Length-Strength Equations

Table 9 is given to enable a comparison to be made between the so-called reduced equations in the form of Equation (2), showing the length-strength relations.

TABLE 9.

Type of Column.	Equation.
Cast iron	$U = 1.000 - 0.0055\,\frac{L}{D}$
Structural steel and concrete	$U = 1.000 - 0.0126\,\frac{L}{D}$
Structural steel	$U = 1.000 - 0.0139\,\frac{L}{D}$
Concrete, no reinforcement	$U = 1.000 - 0.0171\,\frac{L}{D}$
Spiral reinforced concrete, no rods	$U = 1.000 - 0.0173\,\frac{L}{D}$
von Emperger	$U = 1.000 - 0.0179\,\frac{L}{D}$
Spiral and rod-reinforced concrete	$U = 1.000 - 0.0183\,\frac{L}{D}$
Concrete reinforced with rods	$U = 1.000 - 0.0265\,\frac{L}{D}$

SECTION II.—THE INTERNAL BEHAVIOR OF SPIRALLY REINFORCED COLUMNS

1.—Source of Strength

That the ultimate strength of the spirally reinforced column is not due to the spiral alone, the concrete being considered as a substance offering no resistance and, therefore, acting as a perfect fluid, can be proved as follows: The longitudinal column stress and that in the spiral in this case

would be equal, respectively, to the hydraulic pressure in a thin cylinder and the lateral tension in the cylinder walls, the percentage of the volume of the cylinder walls equalling that of spiral reinforcement. The equation giving the relation between the lateral cylinder stress and the hydraulic pressure, transposed to meet the present requirements, gives Equation (33):

$$p = \frac{p_s\, r_s}{2} \qquad (33)$$

in which,

p = the longitudinal stress in the concrete;
p_s = the stress in the spiral; and,
r_s = the ratio of spiral reinforcement.

For a 2% spiral reinforcement of the ultimate strength of 150 000 lb. per sq. in., which values are those to give the largest value to p in Equation (33) for the materials in any or all the test columns examined in Section I, the column stress, p, is found to be 1 500 lb. per sq. in. It is shown, subsequently, that ultimate or preferably maximum column stress occurs when the spiral is stressed at most to its yield point. For a 2% spiral reinforcement stressed to a yield point of 115 000 lb. per sq. in., again values taken from the same group of columns being those to give to p its greatest value, the column stress, p, is found to be only 1 150 lb. per sq. in. These computed values for the column stress are only a fraction of the ultimate stresses actually developed in columns of this type; therefore, the strength of the column is not developed solely by the spiral reinforcement. (See Table 6.) As a corollary, it might be stated that the concrete is not valueless at maximum column load, that it is not completely destroyed, and that it does not act merely as a perfect fluid transmitting the compressive forces to the spiral reinforcement.

2.—Mathematical Equations

Any equation for the spirally reinforced column can be derived from the following five fundamental equations:

$$e_r = \frac{p}{E_c} - \frac{2}{m} \times \frac{p_l}{E_c} \qquad (34)$$

$$e_l = \frac{1}{m} \times \frac{p_l + p}{E_c} - \frac{p_l}{E_c} \qquad (35)$$

$$\frac{r_s}{2}\, p_s = p_l \qquad (36)$$

$$e_l = e_s \qquad (37)$$

$$p_s = e_s\, E_s \qquad (38)$$

in which,

E_c = Young's modulus for concrete (unrestrained);
E_s = Young's modulus for steel (30 000 000 lb. per sq. in.);
p = the longitudinal stress in the column (concrete);

e_r = the longitudinal strain in the column (concrete), the subscript, r, denotes that the strain is for a laterally (by spiral) restrained concrete, and is not equal to the strain as given by the equation, $p = eE_e$;

p_l = the lateral stress in the column (concrete);

e_l = the lateral strain in the column (concrete);

p_s = the stress in the spiral reinforcement;

e_s = the strain in the spiral reinforcement;

r_s = the volume ratio of spiral to concrete within spiral (the percentage spiral reinforcement times 0.01); and

$\frac{1}{m}$ = Poisson's ratio for concrete (unreinforced), that is, the ratio of lateral to longitudinal strain.

The symbol, E_c, requires some explanation. By definition,

$$E_c = \frac{p}{e} \qquad (a)$$

in which, e is the strain resulting in concrete stressed to the intensity, p. E_c is not a constant but varies with the intensity of p. For values of e equal to or less than the ultimate strain of laterally unrestrained concrete, E_c can be determined directly from the equation, $E_c = \frac{p}{e}$, by using observed values of p and e of unreinforced concrete, a method becoming inoperative beyond the ultimate strain of concrete. The value of E_c can also be obtained from Equation (34) for all strain values, both less than and more than the ultimate strain of unreinforced concrete. Note must be made that the value of E_c obtained by the two methods outlined does not correspond for the identical values of the strain, e, and also that the quantity, E_c, as ordinarily understood as the ratio of stress to strain, is an imaginary quantity for values of e beyond the ultimate strain of unreinforced concrete. In both Equation (34) and Equation (a), E_c has the same 1:1 relationship with p.

Equations (34) and (35) are derived from the study of the strains in a homogeneous material under stresses along the three co-ordinate spatial axes.*

If r_s is the percentage of spiral in a column, and a cut is made through the axis of the column, the ratio of the areas of exposed steel and concrete in the longitudinal cross-section is $\frac{1}{2} r$. The total tension in the spiral must equal the total compression in the concrete, or, $A_s\, p_s = A_c\, p_l$. Therefore, $\frac{A_s}{A_c p_s} = p_l$. As $\frac{A_c}{A_s} = \frac{2}{r_s}$, therefore, $\frac{1}{2} r_s\, p_s = p_l$, which is Equation (36).

The spiral is rigidly fixed to the concrete, and, therefore, any strain in the concrete must be accompanied by an equal but opposite strain in the spiral; therefore, $e_l = e_s$.

Equation (38) is the simple stress-strain relation for steel. This is nearly a constant to the yield point.

* "Strength of Materials", by Arthur Morley, Chapter I.

A.—Derived Equations

Substituting in Equation (34) the value of p_l given in Equation (36), there is obtained:

$$p = e_r\, E_c + \frac{r_s}{m}\, p_s \qquad (39)$$

Further elimination of variables gives,

$$p = \frac{m e_r\, E_c \left(m \left[\frac{2}{r_s\, n} + 1\right] - 1\right)}{m^2 \left(\frac{2}{r_s\, n} + 1\right) - (m + 2)} \qquad (40)$$

and,

$$p_s = \frac{2}{r_s}\, \frac{m e_r\, E_c}{m^2 \left(\frac{2}{r_s\, n} + 1\right) - (m + 2)} \qquad (41)^*$$

3.—Application for Equations

Equation (39) shows that the strength of the spirally reinforced concrete is due to two practically independent quantities, the first of which, $e_r\, E_c$, is a function of the concrete, and the second, $\frac{r_s}{m}\, p_s$, a function of the stress in the spiral and Poisson's ratio, all these values being those at the maximum column stress. Equation (39) can be written:

$$p = (e_r\, E_c)_m + X + \frac{r_s}{m}\, p_s \qquad (42)$$

in which $(e_r\, E_c)_m$ is the ultimate or maximum strength of the unreinforced concrete, and, therefore,

$$e_r\, E_c = (e_r\, E_c)_m + X$$

At the ultimate strain of unreinforced concrete, approximately 0.0015 in. per in., $e_r E_c = (e_r\, E_c)_m$. An increase in strain and load beyond this point increases p. In what manner does X vary? This can be determined from Equation (39) by using known and substituted values of, $\frac{r}{m}\, p_s$, in which,

$p_s =$ 150 000 lb. per sq. in.;
$r = 0.02$; and
$m = 2$ (the minimum possible).

All these are values to give the greatest value to $\frac{r}{m}\, p_s$, which becomes $\frac{150\,000 \times 2}{2 \times 100} = 1\,500$, and which is the maximum that $\frac{r_s}{m}\, p_s$ can assume. It is probably, as the criterion, larger than the value which it actually does assume. Again, as it can be proved that the spiral at maximum column stress

* In "Materials of Engineering" Johnson has developed a somewhat similar equation for p_s. This equation has been commonly accepted, but it contains one indeterminate quantity, and checks poorly with experiments. Equation (41) gives values which check closely with experiments.

is at most stressed to the yield point, using yield-point values which give p a maximum value, 1 150 lb. per sq. in. is obtained for $\frac{r_s}{m} p_s$. These values are far below the actual increase in strength due to the spiral for a 2% reinforcement and in the same manner can be proved proportionally less for smaller percentages of reinforcement. (See Table 6.) For example, concrete of a strength of 3 000 lb. per sq. in., reinforced with 2% of spiral, would have its strength increased approximately 3 400 lb. per sq. in., according to Equation (23).

Therefore, X must be positive, as,

$$(e_r E_c)_m + \frac{r_s}{m} p_s < p,$$

and $e_r E_c$ must increase beyond the value $(e_r E_c)_m$. The strength of the spirally reinforced column, therefore, must be due to some inherent quality of the concrete which permits it to be compressed uninjured, when spirally reinforced, greatly in excess of the ultimate strain of unreinforced concrete. Equation (39) shows that the greatest part of the increase in strength caused by spiral reinforcement is due to the increase in $e E_c$ over the value $(e E_c)_m$, and that this part of the increase is practically independent of Poisson's ratio and the stress in the spiral. The magnitude of $e_r E_c$ for any value of e_r, is independent of the stress in the spiral, but is a function of the percentage of spiral reinforcement, as empirically determined.

Referring again to the concrete with an ultimate strength of 3 000 lb. per sq. in., reinforced with 2% of spiral and thus increased by 3 400 lb. to the value of 6 400 lb. per sq. in., the following calculations can be made. The largest possible value of $\frac{r_s}{m} p_s$, in Equation (39), is 1 150 lb. per sq. in. The value of e_r at ultimate column strain, approximately 0.0035 in. per in., compares with the value of e_r at the strain of unreinforced concrete of 0.0015 in. per in. From this, it may be noted that the variable, E_c, is much lower at the ultimate column strength than at that of unreinforced concrete.

The lateral strains for a spirally reinforced concrete are shown by extrapolation of direct experimental measurements to be between 0.0006 and 0.0030 in. per in. (See Section II, 4 D.) This is equivalent to stresses of 18 000 and 90 000 lb. per sq. in., respectively, in the spiral. By Equation (38), the maximum increase in column strength due to 2% of spiral reinforcement (that is, the second term of Equation (39), $\frac{r_s}{m} p_s$), stressed to 90 000 lb. per sq. in., is 900 lb. per sq. in., which is the maximum increase in strength due to spiral alone for the group of columns analyzed in Section I. The maximum increase due to the spiral alone, corresponding to a spiral stress of 18 000 lb. per sq. in., is 180 lb. per sq. in. This is a variation of only 720 lb. per sq. in., due to the maximum difference in spiral stress at maximum column load.

The increase in strength of spirally reinforced concrete, therefore, is practically independent of the strength of the spiral reinforcement or the

stress in it at the instant of maximum or ultimate column load, as the maximum variation in the stress in the spiral causes a maximum variation in column strength of only 360 lb. per sq. in. which is approximately 6 per cent. If the stress in the spiral, at ultimate column load, is between 18 000 and 38 000 lb. per sq. in., the latter being the lowest value of the yield point of the spiral, variation in the strength of the spiral in the particular column would not affect the maximum strength of the column, as all spirals would be capable of being stressed to at least 38 000 lb. per sq. in. If the concrete compressed sufficiently to stress the spiral beyond 38 000 lb. per sq. in. then, from Equation (38), it can be seen that the column strength would be slightly influenced by the variation in yield point and consequent variation in ultimate strength of the spiral.

In the group of columns tested at the University of Illinois, however, the increase in strength due to the spiral reinforcement was independent of the yield-point stress of the spiral. This would indicate that, for stresses of 18 000 lb. per sq. in. or more, in the spiral, which is the spiral stress range in which column failure occurs, the rate of decrease in $e_r E_c$ is algebraically as large as that of $\frac{r}{m} p_s$, or, if $e_r E_c$ did not decrease more rapidly than $\frac{r}{m} p_s$, that all the stresses in the spirals for this group of columns at maximum column load were under the lowest yield point of the group of spirals. In this latter case, the strength of the column would not be influenced by the variation in the yield point of the spirals used. This is within the limits of possibility, although very improbable.

4.—Properties and Behavior Under Load

A.—Poisson's Ratio

Poisson's ratio, $\frac{1}{m}$, is that of the longitudinal to the lateral strain in a material, and, therefore, may be written as $\frac{e_h}{e_l}$. For a perfect fluid, for which Poisson's ratio is a constant and also has the greatest possible value, $\frac{d\ e_h}{d\ e_l} = 0.5$ and $\frac{\int d\ e_h}{\int d\ e_l} = 0.5$. Poisson's ratio cannot be greater than 0.5, otherwise a material could increase in volume due to compressive stress, which is absurd, and leads to a direct contradiction of the principle of the conservation of energy.

Poisson's ratio is not $\frac{d\ e_h}{d\ e_l}$, but is $\frac{\int d\ e_h}{\int d\ e_l}$. As for all solids, the initial value of $\frac{e_h}{e_l}\left(\frac{1}{m}\right)$ is much less than 0.5, the value of $\frac{1}{m}$ at the failure of the material, $\frac{\int d\ e_h}{\int d\ e_l}$, even if at that point the material had become a perfect fluid, would be less than 0.5.

For spirally reinforced concrete, Poisson's ratio is smaller than it is for unreinforced concrete. To the ultimate strain of unreinforced concrete the

spiral stress, as obtained both by measurement and by application of Equation (41), is at the most 10 000 lb. per sq. in. Equation (36) shows that 2% of spiral, when stressed to this value, produces a lateral stress in the concrete of 400 lb. per sq. in. (and for lesser percentage of spiral, a lesser effect). Equation (34) shows that the longitudinal compression is reduced 10% by the 2% of spiral reinforcement, and Equation (35) shows that the lateral expansion is reduced 50% by the reinforcement, the net result being a reduction of Poisson's ratio to five-ninths of its unrestrained value.

Several of the longitudinal and lateral strain readings for columns of the groups tested and described in *Bulletin No. 10* of the University of Illinois and *Bulletin No. 466* of the University of Wisconsin, show the value of $\frac{1}{m}$, computed from readings on identical columns, to increase with the increase in stress, and to show, among possible values, a percentage of such high final values as 0.7, 0.85, etc., and apparent contradiction to the previous statements. On examination of the method of making these measurements, together with the impossibility of measuring exactly Poisson's ratio for a non-homogeneous material such as concrete, the discrepancy between theory and fact vanishes. The lateral strain was measured at one or two places on the column; the longitudinal strain was measured for the whole length of the column. Consequently, the longitudinal strain measured was the average for the whole column, but the measurements of lateral strain showed the value for only one or two particular points on the length of the column. The two quantities were not measured, therefore, for the same series of particles of concrete, only one or two particles being common to the two measurements. If the strains were constant throughout the mass of the concrete, this method would be valid, but with a non-homogeneous substance such as concrete, the variation in strains throughout the mass makes such measurements valueless as, for example, the present values obtained from Poisson's ratio. Although the magnitude of $\frac{1}{m}$ is valueless, the manner in which it varies with variation in load can at least be placed to some use, as it indicates qualitatively, if not quantitatively, the variation of the true value of Poisson's ratio with increasing stress.

In justice to the authors of the two *Bulletins* mentioned, it must be stated that they realized the fallacy of such figures and did not include them in the values which they measured and computed for Poisson's ratio. Professor Talbot mentions the inadequacy of the results. Messrs. McKibben and Merrill include the three astounding values of 0.82, 0.563, and 0.620, in their measurements.

Fig. 7 gives several representative curves for the so-called Poisson's ratio, derived from the one-place lateral measurements. It can be seen that $\frac{1}{m}$ always increases with increase in load, but in no general manner. Some curves are nearly straight lines, whereas others are similar to the stress-strain curves for spirally reinforced concrete, having a rather sudden change of direction, with a long continuation of the curve.

B.—Ultimate Strain for Spirally Reinforced Concrete

Professors Withey and Talbot have made numerous direct measurements of the strain of spirally reinforced concrete columns, several representative curves being given in Fig. 8. For obvious reasons, the ultimate strain cannot be determined precisely in this manner. The strains are very small linear measurements, even for such comparatively large strains as occur at the failure of spirally reinforced concrete. The shell has been sheared, and the surface

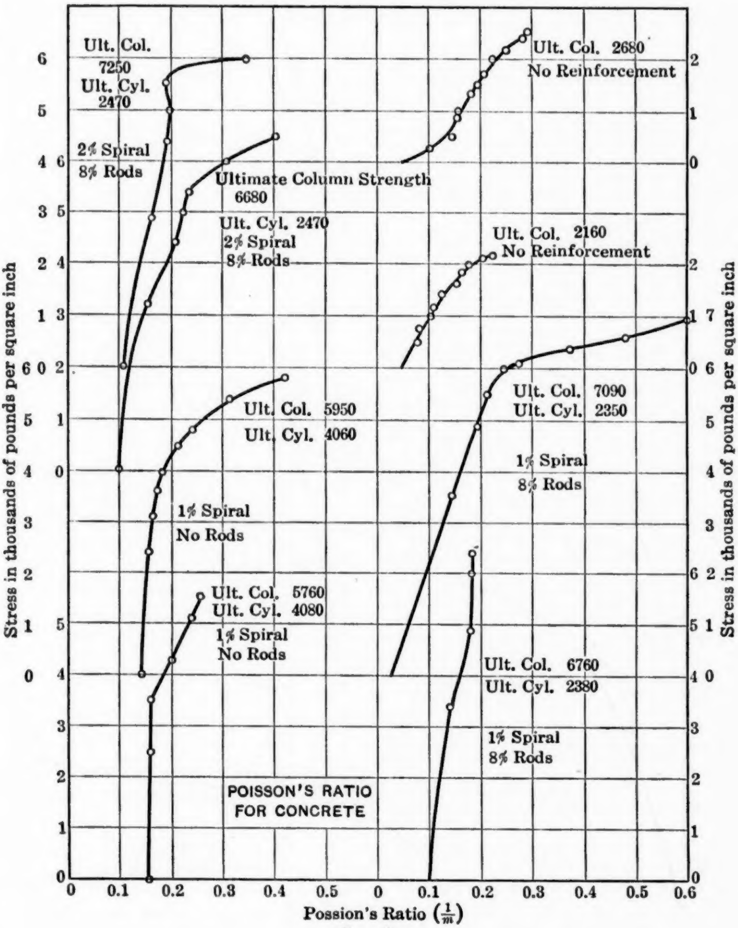

FIG. 7.

concrete is in a state of spalling. Such conditions are not conducive to good results. Nevertheless, observations by such excellent technicians at least show the stress-strain relations to and near failure and give good indications of column behavior at the point of failure. Several observations show clearly the strain at which maximum (ultimate) stress was developed. The stress at that point was a true mathematical maximum, the stress values for points on

curves passing through the point being less than the stress value at the point, for all values of the strain lesser and greater than the strain value at the point. In these cases, the strain was carried a considerable distance beyond the strain at maximum load. It may or may not be that the cause of the maximum

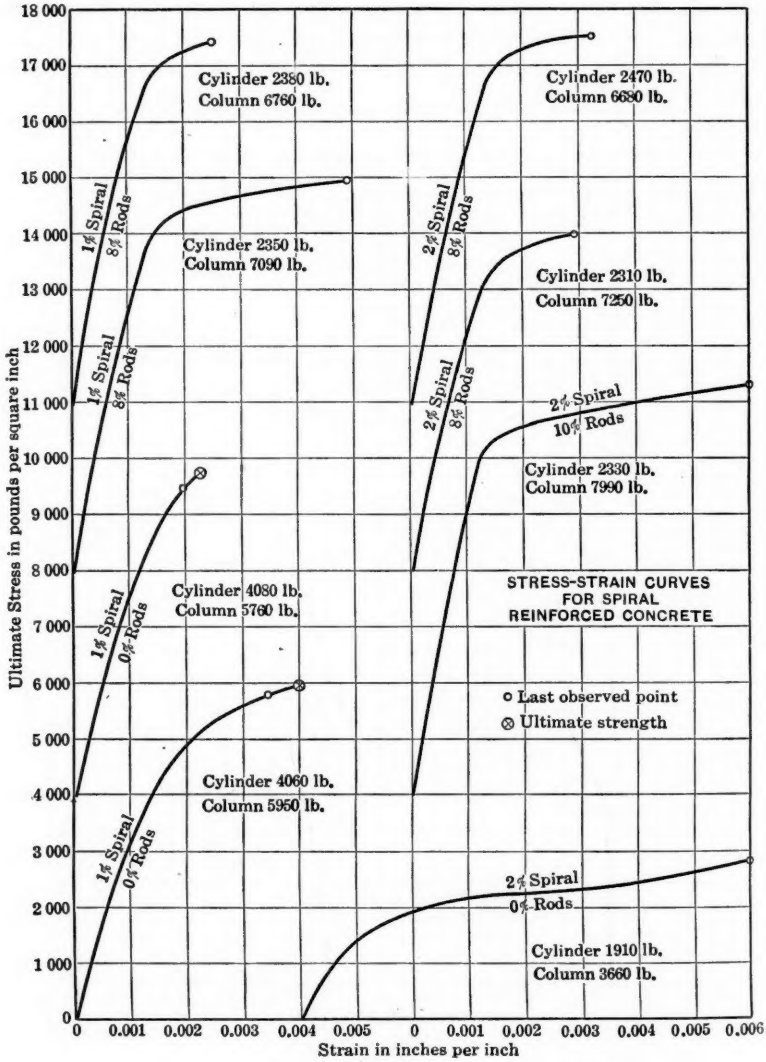

Fig. 8.

stress point in the stress-strain curve was the stressing of the spiral to the yield point. Referring to Equation (39), it will be noted that the second term in $\frac{r_s}{m} p_s$ would remain practically constant, as $\frac{1}{m}$ is not capable of any great increase, which it would have to undergo noticeably to increase the whole

term, and p_s would remain constant, as the yield point had been reached. It was shown that $e_r E_c$ increases considerably beyond the value at the ultimate stress of unreinforced concrete; yet for the part of the stress-strain curve at the stage where the stress is practically constant for increasing strain, $e_r E_c$ may decrease. In the case of a decrease, the reaching of the yield point of the spiral would show plainly as a maximum on the stress-strain curve of the reinforced column. The extrapolated stress-strain curves give values of the ultimate strain ranging between 0.00165 and 0.015 in. per in.

The set of columns of the von Emperger type described in the last part of Section I give an independent means of checking these figures. The cast-iron reinforcement in these columns is not stressed to its ultimate strength at the maximum column strength. The strength of, or rather the load borne by, the cast iron can be computed as the difference between the ultimate strength of the reinforced concrete and the ultimate column load. Having the cast-iron, stress-strain curve, the strain is obtained directly for the ultimate column strength, which is the ultimate strain for spirally reinforced concrete. The strain calculated in this manner for the average of the several columns proves to be 0.0034 in. per in., agreeing with the values obtained by extrapolation of direct measurements of the strain.

C.—Stress-Strain Relations for Spirally Reinforced Concrete

To determine the true stress-strain curve for reinforced concrete, the column should be tested without the protective coating or shell. If the shell is initially present, the course of the loading presents two phases which are interfluent, commencing at the point where the shell begins to fail and continuing until it has been completely sheared off. The total loss of the shell rarely occurs, parts of it remaining even at maximum load. In the initial phase, the column and the shell are integral, but in the final stage the shell has sheared off in whole or in part, and the column has an indeterminate diameter, varying from that of the spiral reinforcement as a minimum to that of the original column as a maximum. Thus, the diameter of the column changes during load, and it is impossible to obtain the stress-strain curve from the load-strain curve.

The load-strain curve of the column with the initial shell is of the general shape shown by Curve *A* in Fig. 9. If the shell was instantaneously destroyed at the ultimate strain of unreinforced concrete, the load-strain curve for the column would have the shape of Curve *C*, Fig. 9. This type of curve would result in the case of instantaneous failure of the shell, because after such failure, the core would be supporting the load that previously had been supported in part by the shell. The net result of such failure would be to increase the strain without the application of additional load, resulting in an action similar to that of steel at the yield point. From the load-strain curves obtained from tests on columns of this type, which do not show any indication of the marked changes of direction, it is proved that the shell fails gradually.

If a column without a shell is tested, a curve of the general form of Curve *B*, Fig. 9, will result, the break in the curve being more sudden, and with no

such sweep as that shown in the region, *a* to *b*, Curve *A*, Fig. 9. This wide bend of the stress-strain curve has been seen as an indication of the weakness of this type of column. The stress-strain curve for the core of the column, however, shows a much straighter curve to what might be termed the yield point.

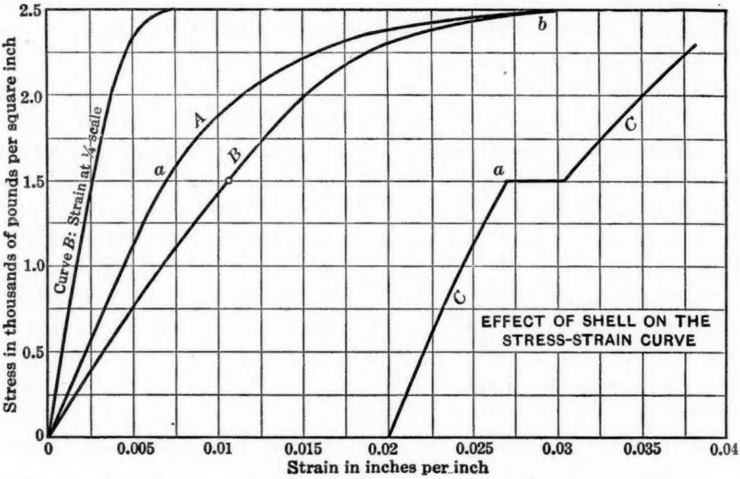

FIG. 9.

D.—Lateral Strains

Extrapolation of measurements by Professors Withey and Talbot give values of the ultimate lateral strain for spirally reinforced concrete, that lie between the limits of 0.0006 and 0.0030 in. per in. The greatest measured lateral strain is well before the point, *b*, Fig. 10, or the end of the yield-point elongation for steel wire. The lateral strains for this type of column to the failure of the concrete core are, of necessity, equal to the strains in the reinforcing wire. This

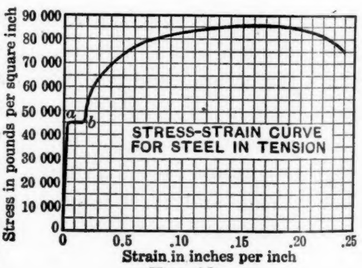

FIG. 10.

shows that in every case the spiral was stressed at the most to yield point, and, frequently, much less, when column failure or maximum load occurred. The failure of the column is due to the failure of the concrete, and not to the failure of the spirals.

E.—Method of Failure of the Spirally Reinforced Column

From Equation (40) and a representative column stress-strain curve, the values of E_c for strains beyond the ultimate strain of unreinforced concrete can be calculated with fair accuracy. The unknowns in the equation are $\frac{1}{m}$ and E_c. All the other quantities in the equation, either directly or in-

directly, have been measured with accuracy sufficient for the purpose. The value that $\frac{1}{m}$ assumes at the ultimate strain of unreinforced concrete must be between the limits of 0.25 and 0.50. It is theoretically impossible for $\frac{1}{m}$ to be greater than 0.5 and it must be as large as 0.25, as shown by such readings as have been made and by the indication of future action. Any value of $\frac{1}{m}$ within this range, substituted in Equation (40), gives nearly the same value for E_c, the effect on E_c of the variation of $\frac{1}{m}$ between these limits being small. Therefore, E_c is determined with fair accuracy and $\frac{1}{m}$ within a range of 50 per cent.

Using in Equation (9) any conceivable and possible simultaneous set of values for the variables within the range that it has been shown possible for them to have, it is impossible to obtain a computed value for the stress in the spiral reinforcement of more than the yield point. Observations also show that, at the maximum column load, the spiral at most is stressed to the yield point. Assuming $\frac{1}{m}$ equal to 0.5, the value to give the greatest value to p_s, and the values of E_c, or $e_r E_c$, as obtained by Equation (40), it is found that, for a longitudinal strain of 0.01, Equation (41) gives a much lower value to p_s than was determined by Equation (40) which strain is known to be true. Substituting 0.01 as the value of the longitudinal strain, and every measured or possible value for the remainder of the variables in Equation (9), always leads to the same result, that is, a calculated value of p_s much less than the known measured value of p_s or its value calculated from Equation (40).

This is proof that it is impossible for a material having any rigidity whatsoever to stress the spiral beyond the yield point, no matter what that value may be, if it is more than a minimum of 35 000 lb. per sq. in. It is known, however, that compression of reinforced concrete columns eventually ends in the breaking of one or several of the spirals. This means that the load is transmitted through a concrete that has properties different from the original whole concrete. The change in the concrete may be accounted for in two ways, as follows:

a.—The concrete becomes a perfect fluid, transmitting the longitudinal compressive stresses directly to the spirals. If this hypothesis was true, the stress-strain curve of the column would show a great drop in the load at the point at which the concrete became fluid. Observations do not show such a drop, thus proving that this hypothesis is incorrect and that the concrete does not become a fluid.

b.—The concrete of the column shears or crumbles, shifting the load more directly on the spirals. The friction between the sheared or crumbled surfaces, together with the tension stresses in the spiral, would explain the high column

stress after the concrete had ceased to be intact. Such shearing would result in local stressing of the spirals in the shear plane. The breaking of several turns of the spirals at points directly under each other bears out the hypothesis. This is the only possible explanation of the failure of spirally reinforced columns. It is to be noted that the breaking of the column spirals in tension is only an inconsequential anti-climax and is without importance.

Observations on the tests of the von Emperger columns showed that the spirals failed after the maximum load had been applied, the cast iron breaking finally. Cast iron fails at a strain of about 0.01 in. per in. To cause failure of the spiral, a strain of 0.25 in. per in. is necessary. For a perfect fluid the longitudinal strain necessary to produce a strain in the spiral of 0.25 would be 0.5. As concrete, even at the ultimate spirally reinforced strain, is not quite a perfect fluid in the limiting case, the longitudinal strain necessary to cause the failure of spiral in tension would be at least 0.5. The ultimate strain, however, of cast iron, which is considerably greater than that of spirally reinforced concrete, is only 0.01 in. per in.

That the failure of the spirals in a column is due to an initial shear failure in the concrete is borne out by the fact that test columns after failure show a general shear and crumbling in the plane of the tension break in the spirals. At all points throughout the axis of the test column, the concrete was strained beyond the ultimate of unreinforced concrete, although, after testing the concrete column, the concrete will be sound, except where the spirals have failed.

The two columns, described in *Bulletin No. 466* of the University of Wisconsin, of strong concrete (about 4 900 lb. per sq. in.), show clearly by their stress-strain diagrams that failure occurred in the concrete, stress-strain measurements being made to maximum load. These curves are almost straight lines, and show no inclination to change direction before failure like the columns of lower strength, similarly reinforced. The stress computed from the measured strain of the spiral gives only about 12 000 lb. per sq. in. for a load just under the maximum. It appears impossible that the stress-strain curve should suddenly change its direction at right angles, or nearly so, just prior to failure. This would have to be the case if the spirals were the cause of the failure. Professor Withey concludes from observations that the failure of these columns was due to the failure of the concrete.

Modern compression testing machines apply load by means of increasing strains. Consequently, in a test of a column to destruction, the course of the phenomena is altogether different in some phases from that which would occur if the load was applied directly by means of weights. One great difference is in the time. It has been shown that in the spirally reinforced column the concrete fails first and shifts the load on the spirals. The means of increasing the load in the testing machine is the application of more strain, which is an exceedingly slow process, especially for the very large strains necessary to cause breaking of the spiral in a column of the type under consideration. A period of perhaps 15 min. would be required to accomplish in the testing machine what would occur instantly under a gravity load. The load would

be shifted to the spirals the instant the concrete failed, the column shortening rapidly, the spirals failing, and the column, as a whole, collapsing.

5.—Summary

1.—The increase in the strength of concrete due to spiral reinforcement is independent of:

(*a*) The yield point of the spiral when that point lies within the limits of 38 000 and 115 000 lb. per sq. in.

(*b*) The ultimate strength of the spiral when this value is between 60 000 and 150 000 lb. per sq. in.

2.—At maximum column load, the spiral is stressed to some value between limits of about 18 000 and 90 000 lb. per sq. in., or to the yield-point stress if it is less than 90 000 lb. per sq. in.

3.—The failure of the spirally reinforced concrete column is caused by failure in the concrete itself, by crushing, shearing, or a combination of both. As a corollary, the failure of the spirals in tension is not related to the column failure, but is a phenomenon beyond this point.

4.—The increased strength of the column is due to the increased strength of the concrete itself, because of the lateral restraining action of the spiral on it.

5.—There is an upper limit to the strength of the concrete that can have its strength increased by the use of spiral reinforcement. Beyond about 4 000 lb. per sq. in., the increase in strength due to spiral reinforcement diminishes rapidly with that of the unreinforced concrete, until at about 4 900 lb. per sq. in., the strength of the concrete is not appreciably increased by the addition of spiral reinforcement. Theoretical (by Equation (40)), and also actual measurements show the increase in the strength of 4 900-lb. concrete to be approximately 5 per cent.

6.—Concrete has its strength increased (for a column of nine diameters) by spiral reinforcement, as given by the formula:

$$f_{os} = f'_c\,(1 + 0.573\,r_s)$$

in which,

f_{os} = the ultimate strength of the reinforced concrete;
f'_c = the ultimate strength of the unreinforced concrete; and,
r_s = the ratio of spiral reinforcement.

For a column of any length, the strength of the reinforced concrete is:

$$\frac{P_m}{A} = 1.27\left(1.00 + 0.573\,r_s\right)\left(1.000 - 0.0173\,\frac{L}{D}\right) f_c$$

in which $\frac{L}{D}$ is the ratio of length to diameter.

7.—Longitudinal reinforcing rods add definite strength to the column, approximately 32 000 lb. per sq. in. of rod cross-section, the value depending on the length of the column. The strength including that added by the rods is given by the formula:

$$\frac{P_m}{A} = 1.20\left[(1 - r_r)(1.00 + 0.573\,r_s)\,f_c + 36\,500\,r_r\right]\left[1.00 - 0.0183\,\frac{L}{D}\right]$$

SECTION III.—RELIABILITY

The knowledge of the reliability of the physical properties of a material is indispensable in engineering design, being equally as important as the knowledge of the average values of such properties. The most important of these properties are ultimate stress, yield-point stress, or other salient property of the stress-strain relationship, hardness, resilience, etc. The only properties of the material or structural element discussed in this paper are those of strength, namely, elastic limit, yield point, ultimate stress, and other possible salient features of the stress-strain relationship, endurance limit (stress), continued load stress, resilience, etc. Other physical properties, such as the resistance to wear, are unimportant in this consideration.

The reliability not only of the properties of materials but also of built-up units is of the utmost importance, as that of the property of the final structure or structural element is the object sought. The reliability of the property of the material is of importance only because of its influence on that of the property of the structural element into which the material enters.

The allowable, safe working stresses, or the factors of safety, are surely functions of the reliability of some one of the strength functions of a material or structural element, and should be deduced with a proper consideration of the affect of this reliability. Committees appointed to determine the safe and allowable working stresses always give great weight to the reliability, or more properly, lack of reliability, of the strength of materials, especially for such materials as cast iron and concrete, in which the reliability is very low. If not explicitly considered, it is unconsciously taken into account with the instinctive knowledge which is, or should always be, possessed by those whose duty it is to draw up and revise the specifications for working stresses. Nevertheless, the reliability is never given the consideration it deserves. Even when cognizance is taken of the reliability and an attempt is made to allow for the lack of it, such attempt is made in the crudest manner. Objection will be made to a certain stress on the ground of the unreliability of the material, which results in a reduction in the allowed working stress to provide for the poor reliability. Although accurate results sometimes are obtained by thus using the judgment and intuition of those long familiar with the materials, this method is equally as likely to result in huge errors. Wherever possible it is being supplanted by the correct method which omits the personal equation and permits the results to be obtained mathematically from measurements made by any of a skilled number of experimenters.

Technical engineering literature contains frequent reference to reliability, but always in the vaguest manner; it is never expressed numerically or even in definite terms that would afford a means of comparison between the reliabilities of the strengths of two materials. That the strength reliability of steel or fabricated steel structural elements is high, and that the strength reliability of cast iron and concrete is low, is a true and obvious statement. It is, however, capable of limited application, especially in the determination of the influence of the reliability on the working stresses to be permitted. What

of the relative reliability of two steels of different composition and heat treatment, or of two materials that gave strengths for test specimens of 100, 140, 150, 160, and 200 for five specimens of the one and strengths of 110, 120, 150 180, 190 for five test specimens of the other, the average of both being the same? Although reliability may readily be intuitively perceived in a general sense, yet when refinements enter, the determination by intuition can readily be seen to fail completely.

How is the strength reliability of materials exhibited? Is cast iron an unreliable material because of the suddenness and violence of its failure? The Quebec Bridge was built of the most reliable material known, yet, between the first indication of failure and complete failure, so little warning was given that it was valueless, either to prevent the failure or to enable any of those on the bridge to escape. It is true that this was an unfinished structure, but had failure of the finished structure occurred, due to innumerable possible causes, it would have been just as sudden. Spiral and rod reinforced concrete is regarded as having more strength reliability than unreinforced concrete. Yet, subsequently, it is shown that spiral and rod reinforced concrete and unreinforced concrete have practically the same reliability. Owing to the careless methods of fabricating the concrete and also in checking the strength of the concrete entering the structure, its strength is not always what it should be. For this reason, the longitudinally reinforced concrete may be regarded as more reliable, as only a part of the strength of the column, namely, that due to the concrete, is lowered by the concrete of inferior strength. The testing of spiral and rod reinforced concrete requires a comparatively long time for failure to be effected, even after the first positive signs. The long time necessary to cause failure is the result of the method of application of the load in the modern testing machine. Although the spirally reinforced column possesses much greater resilience and, therefore, can withstand shock much better, the static load that would cause the spirally reinforced column to fail would cause that failure to occur instantly. Evidently, then, the suddenness and violence of the failure or the length of time required for failure to occur in the testing machine is no indication of the strength reliability of the material or structural element.

Reliability is a measure of uniformity, and, therefore, can be measured by means of the variations of the individual specimens from the mean value of the magnitude of whatever property of the particular object that is under consideration. The process of determining the reliability for a specific case theoretically would necessitate the discovery of the law of the manner of the variations. Although research to determine the manner of variation is to be expected, such knowledge is not imperative. The variations may safely be assumed to be identical with those given by the standard error function. From observations of the variations it is now possible to compute, by the theory of probabilities, the value of the property under consideration, which will be exceeded in all cases. This may be stated as the value which will be exceeded only in all but an infinitesimal number of cases. To illustrate the

application, if a material has a mean strength of 2 000 lb. per sq. in., and a reliability number of 0.75, then every specimen made of this material would safely stand a stress of $0.75 \times 2\,000 = 1\,500$ lb. per sq. in. A more detailed discussion and development of the theory is given subsequently.

1.—Physical Measurements

Whether the desired result is a simple measurement of length, or the temperature of the sun, no two observers will arrive at the same measurement of the desired quantity, nor will the same observer be able to obtain in succession the same numerical result.

There is no such quantity as the "true" value, but it may be taken as that value of the quantity determined by the most refined means and instruments possible. Mach, Pearson, and others have proved that all things are relative, therefore, any measurement is relative, being a comparison of the object with a standard. The property of the object measured did not exist—the only reality was the measurement or comparison. The property cannot be separated from the object.

In general, the variations in results are due to possible difference in method, the personal equation, which enters perceptibly when the element of time is operative or a distance must be estimated, etc., difference due to errors in the instruments used, to uncorrected effects of light, temperature, electric and magnetic fields, atmospheric influences, all of which, in most cases, cannot be eliminated, except in the most refined experiments, and also those possible disturbing factors which as yet are unknown.

By making a large number of observations, using several methods, and having observations made by several observers and obtaining the average, obviously, a result much more accurate will be obtained than any single observation in the group. The errors or variation of any result from the true value will as likely be positive as negative, therefore, the sum of a large number of errors is so nearly zero that the difference can be neglected. Therefore, the averaging of a large number of observations automatically eliminates the errors, which is expressed by the statement that the errors are compensating.

In testing concrete, no attempt is made to correct for atmospheric effects other than to keep the air saturated with moisture during the first week after making the specimen, and to keep the specimen at approximately room temperature. The strength of concrete will be affected by the various errors in the machine in which the concrete is tested, and by the method of mixing it. Accidental eccentricity of loading and rate of application of load will cause appreciable variation in the apparent strength. The thoroughness of mixing, percentage of water used, variation in the average size and grading, and the strength of the coarse aggregate, etc., all tend to vary the strength of concrete made from aggregates of identical proportions.

The variations in strength of individual test specimens of any material, or of any composite or built-up structural element, nearly coincide in their mathematical relationship with the errors of observations as given by the well-known "error function" formula. The variation in strength of such a material as

rolled steel is not as apparent as it is in materials such as cast iron and concrete. This strength variation may be viewed as the result of the effort of the internal components, either microscopic or macroscopic, or of the magnitude of the aggregate of concrete, to arrange themselves according to some average scheme; the change in arrangement (the arrangement being produced during fabrication) would cause differences in strength.

The formula describing the manner in which errors occur as theoretically developed a century ago, has been substantiated by repeated experiment for the errors of observation and like cases on which the original hypotheses for developing the formula were based. Experiment also shows that the variation in strength of individual test specimens from the mean or average value of the strengths of those test specimens, follows, as nearly as can be expected from the large magnitude of the errors and the relatively small number of specimens tested, the general equation. The so-called "error function", which mathematically describes the possibility and magnitude of any error is a function of the form:

$$y = \frac{h}{\sqrt{\pi}} e^{-h^2 v^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

in which,

v = the magnitude of the error;

y = the relative probability of the error, x, occurring;

h = a constant, also the intercept or the relative probability of the error being zero.

e and π have the usual significance.

By custom:

$$\frac{h}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-h^2 v^2} \, d\, v = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

or, expressed otherwise, the total probability is arbitrarily established as unity.

Plotting a large series of observations, the abscissas representing the error or variation of the individual specimen or reading from the mean or average value of the observed quantity, and the ordinates the number of observations that were made having the same error equal in magnitude to the value of the ordinate, a curve of the same general contour as that given in Equation (44) is obtained. By a change in the constant to make the integral, $\int_{-\infty}^{\infty} y \, d\, v = 1$, the theoretical curve is nearly obtained, the experimental curve for various reasons not coinciding exactly with the theoretical one. The necessity for an enormous number of observations in order closely to approximate the theoretical curve is usually the main reason for the lack of coincidence.

The value of the constant, h, can also be algebraically determined from the formula:

$$h = \sqrt{\frac{n}{2 \Sigma v^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

in which,

n = the number of observations; and

v = the variation of the individual observation from the mean of the observations.

The variation in the ultimate strength of a material obeys or follows the laws stated, with only a slight general exception. The standard probability curve extends to infinity in the positive and negative directions. That the strength of a material can be less than zero, however, is absurd. In Fig. 11, if D is the average strength of a material, and the strengths of the specimens and the percentage number of specimens are the abscissas and ordinates, respectively, then the curve will be the same as the probability curve between D and infinity, but will be a different function between O and D, of the form shown in the diagram in exaggeration, and is such that,

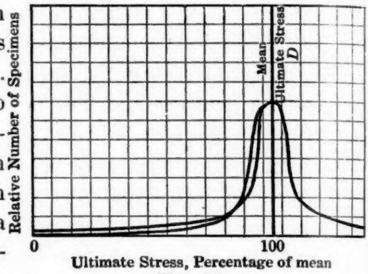

FIG. 11.

$$\int_O^D y\, dv = \frac{1}{2} \qquad (46)$$

The difference between the results obtained from Equation (46) and Equation (44) is small, so that the general or standard formula and probability tables may be used.

The probable error is defined as the value that is as likely to be greater than the real error of the result as it is to be smaller. This can also be expressed: The probability of the error of the result being less than the probable error is equal to the probability of the error of the result being greater than the probable error.

The probable error is determined by the equation:

$$R \equiv p.e. = 0.6745 \sqrt{\frac{\Sigma\, v^2}{n\,(n-1)}} \qquad (47)$$

for the mean of a number of observations; and,

$$R \equiv p.e. = 0.6745 \sqrt{\frac{\Sigma\, v^2}{n-1}} \qquad (48)$$

for a single observation. That is, Equation (47) gives the probable error of the arithmetic mean of several observations, and Equation (48) gives the probable error for any one observation, which may be one of the observations made in determining Equation (47).

The probable error, R, although an inverse measure of the reliability, is capable of no application in the present consideration, and further consideration of the properties of the error function is necessary. The fundamental equations:

$$R \equiv 0.6745 \sqrt{\frac{\Sigma\, v^2}{n-1}} = \frac{0.4769}{h} \qquad (49)$$

$$t = h\, v_0 \qquad (50)$$

from which,

$$t = 0.04769 \frac{v_0}{R}, \text{ or } v_0 = \frac{t\,R}{0.04769} \qquad \text{(51)}$$

in which, v_0 is any value of the error or variation connected by a 1:1 relation with R by t, a variable introduced to simplify probability calculations, probability tables giving values of t for any desired probability.

P_t, the probability that the error, v_0 corresponding to the value, t, will not be exceeded in a negative sense, is given by the equation:

$$P_t = \frac{1}{\sqrt{\pi}} E\,r\,f\,t = \frac{1}{\sqrt{\pi}} \int_0^t e^{-t^2}\,d\,t$$

$$= \frac{1}{\sqrt{\pi}} \cdot \left[t - \frac{t^3}{3} + \frac{1}{2\,!} \cdot \frac{t^5}{5} - \frac{1}{3\,!} \cdot \frac{t^7}{7} + \ldots \right] \qquad \text{(52)}$$

It can be readily proved that:

$$[1 - P_t] = \frac{1}{\sqrt{\pi}} Erfct = \frac{1}{\sqrt{\pi}} \int_t^{\infty} e^{-t^2}\,d\,t$$

$$= \frac{1}{\sqrt{\pi}} \cdot \frac{e^{-t^2}}{2\,t} \cdot \left[1 - \frac{1}{2\,t^2} + \frac{1\,.\,3}{2^2\,t^4} - \frac{1\,.\,3\,.\,5}{2^3\,t^6} + \ldots \right] \qquad \text{(53)}$$

which is the probability that the error, v_0, will be exceeded.

Values of t can be determined for any assigned value of P_t or $(1 - P_t)$, by means of Equations (52) and (53). From t determined by Equations (52) and (53), can be found the value of v_0, the error whose probability of being exceeded is $(1 - P_t)$.

If x_m is the mean value of the variable, and x_0 the value of the variable which is v_0 in error,

$$x_0 = x_m - v_0 = x_m - 2.10\,t\,R$$

$$x_0 = x_m \left[1.00 - \frac{2.10\,t\,0.6745}{x_m} \sqrt{\frac{\Sigma\,v^2}{n-1}} \right]$$

$$= x_m \left[1.000 - 1.415\,t \sqrt{\frac{\Sigma \frac{v^2}{x^2_m}}{n-1}} \right]$$

$$= x_m \left[1.000 - 0.01415\,t \sqrt{\frac{\Sigma \frac{v^2\,.\,100^2}{x^2_m}}{n-1}} \right]$$

$$= x_m \left[1.000 - 0.01415\,\mathrm{t} \sqrt{\frac{\Sigma\,v^2_p}{n-1}} \right]$$

As $v_p = v$, expressed in percentage of $x_m = \frac{v}{x_m} \times 100$.

It is now necessary to determine the value of t so that the possibility of v_0 being exceeded is infinitely small. The factor, $1.000 - 0.01415\,t \sqrt{\frac{\Sigma\,v^2_p}{n-1}}$, for this value of t will be termed, N_r, the reliability number. N_r can also be

defined as the ratio of the value of the variable which will always be exceeded to the mean value of the variable.

Fig. 12 gives the graph of $1 - P_t$ and t within the range of values necessary in this analysis. Mathematically, t should have the value of infinity to give $1 - P_t$ the value of zero, but the selection of the value of t for use depends on the final application of N_r, the reliability number, this application being the determination of the safe working stress. It is known that there are certain causes producing the variation in the variable the N_r of which is to be determined. If this variable is the ultimate strength of concrete or reinforced concrete, severe loss of strength due to these causes greatly aggravated, such as voids, draining of the fine aggregate and cement to leave only the coarse aggregate in place, will result in an appearance of the finished product, that will cause it to be rejected. It is not necessary, therefore, to assume too minute a probability. The minuteness of the probability of the strength or value of the variable being inferior to that given by the substitution of the assumed value of t need be only relatively minute. To illustrate the impossibility of assuring a safe value for N_r, no mathematical equation can assure that, in a unique case, all the cement may not be omitted from the mix, or a substance incapable of producing any strength whatever be substituted. These possible but very improbable accidents give a concrete of zero strength, yet their occurrence can be made to be without ultimate danger by taking test specimens from every batch of concrete.

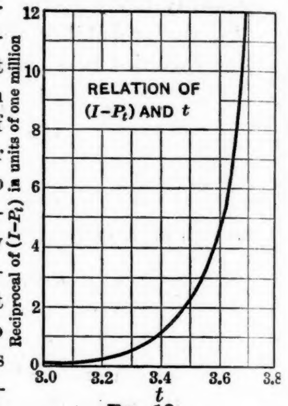

FIG. 12.

In the present consideration, no direct conception of the relative magnitude of the order of millions, or of the necessity for relatively absolute safety, is possible. Some comparison or other means must be used to bring the problem within the grasp of understanding. Such a comparison is available in the safety from death of the normal man. Calculations show that the chances that a human being will die within 10 min. of any predetermined instant is 1 in 10 000 000. This probability will be accepted as perfect safety and is synonymous with infinity. The problem may also be regarded as the determination of the value of the reciprocal of the probability $(1 - P_t)$, which is infinity. A similar problem is that of the number of repetitions of stress that may be considered as infinite, or which will be sufficient to cover the number of repetitions received by a component material in the life of a machine. Values of infinity have been suggested* for several machine and structural elements varying greatly with the use. For steam turbine shafts, the given value is 15 000 000 000, for railroad bridge chords it is 2 000 000.

The probability of 0.000001% will be almost surely accepted as being completely satisfying. Suppose, however, that this probability is assumed as un-

* *Mechanical Engineering*, 1919, p. 731.

necessarily small, that a comparatively larger probability value, say, 0.00001%, 1 chance in 1 000 000, or 1 chance in 500 000, is sufficient. Comparing the values of t (with the reciprocal of which the working stresses are roughly proportional), for these two probabilities, there is:

Probability:

$1 - P_t$	t.
1 in 500 000	3.31
1 in 1 000 000	3.39
1 in 10 000 000	3.68

Only one-tenth the safety is obtained by a decrease of t of $8\frac{1}{2}$%; by an increase of 10% in permissible strength only one-twentieth the safety is obtained. These slight increases in working stresses, with the accompanying large decreases in safety, can only lead to the conclusion that the value of 0.000001%, or 1 in 10 000 000, for the probability should be chosen as the basis of the safe working stress.

2.—Strength of Steel and Determination of Reliability Number

It must be noticed that the variation in strength developed by several specimens is not necessarily a measure of the reliability of the composite element or material in question. When a factor enters, the variation of which causes variation in the strength of the material and there is previous knowledge of the effect of this factor, or ability to control it, then the variation in strength due to the factor is not due to any unreliability of the material itself, and can be separately determined and discounted.

Take the example of steel in tension: The variation in magnitude of the carbon content of steel causes the strength to vary greatly (other constituents and treatment remaining constant). Low-carbon steel has an ultimate strength of about 55 000 lb. per sq. in. High-carbon steel can be manufactured with an ultimate tensile strength of 150 000 lb. per sq. in. If no consideration is given to the carbon content of a group of wires selected at random and tested, the logical conclusion, on the determination of the strength reliability, would be that steel wire is very unreliable. However, if the carbon content is noted, one perceives that the strength is a function of the percentage of carbon in the steel, and by arranging the wires into groups having the same carbon content, it can be seen that, for any group, the strength is very uniform. On determining the reliability number for each group, it is seen that steel is a very reliable material. The treatment that steel receives in fabrication also varies the strength, but for similar carbon content, usually the same treatment is given in the case of reinforcing wire. This discloses the importance of determining, as far as possible, any factors that cause variation in strength, both to eliminate weaknesses that they cause and to eliminate their effect on the apparent reliability.

Table 10 gives the reliability number, N_r, for the various types of columns analyzed in Section I, and also the product of N_r and the ultimate strength, which will be the maximum safe stress of one application of load.

Casual inspection of Table 10 will show what experience has in a broad way proved; steel stands first in reliability, far superior in this respect to concrete or cast iron. The group of columns depending on concrete for their main strength show close agreement in their reliability numbers, which mutually shows that the premises on which the formulas in Section I are based must be correct, and the variation in strength of the column is due to the variation in strength of the concrete, and, therefore, concrete is not made less reliable by the introduction of reinforcement.

TABLE 10.

	r	$0.0772r$	N_r	Product of N_r and ultimate stress.
Structural steel	2.05	0.1559	0.84	*29 400
von Emperger type	2.36	0.182	0.82	†8 200
Structural steel and concrete	4.31	0.332	0.67	‡1 660
Concrete and rods	5.61	0.432	0.57	1 420
Concrete and spiral	5.63	0.434	0.57	§
Concrete rods and spiral	7.00	0.540	0.460	§
Concrete unreinforced	5.83	0.450	0.550	1 370
Cast iron	12.00	0.927	0.073	‖5 100

* Net strength steel in compression equals yield-point stress or about 35 000 lb. per sq. in.

† The von Emperger type will be more fully considered subsequently.

‡ Ultimate stress of concrete × N_r for concrete of 2 500 lb. ultimate strength.

§ Varies greatly with percentage of reinforcement and concrete strength.

‖ Ultimate strength = 70 000 lb. per sq. in.

The great variation in strength of the cast-iron specimens should not be finally indicative of this type of column; it can be explained partly by the fact that the cast-iron specimens tested were regular commercial specimens, as against the carefully prepared laboratory specimens of concrete, and, also, that they were made in various foundries with different grades of iron, the result being a variation in strength due to the use of different materials similar to that caused by different carbon content in steel, previously discussed. This latter affect could have been eliminated by basing the strength of cast iron on small test specimens from the same pour; but, unfortunately, these small specimens were not made. With the advances that have been made in the technique of casting iron columns, there is no need to place all cast-iron columns in the same unreliable category.

The reliability number has been determined from the ultimate strengths for the group of test columns. The determination of that number for the columns of a new structure, however, would necessitate the testing to destruction of a number of the group, which is an impossible commercial procedure. Test cylinders of the concrete composing the columns must form the basis for the determination of the reliability number for the column. The variation in strength of concrete in test cylinders and in a column of unreinforced concrete is not identical. To obtain the N_r for the reinforced column from concrete test cylinders, the ratio of the N_r of test cylinders to columns (reinforced) must be introduced as a factor.

SECTION IV.—ALLOWABLE WORKING STRESSES OR FACTORS OF SAFETY

1.—Introduction

Methods of mathematical analysis of the stresses and strains in any conceivable type of structure have already been sufficiently developed so that, for any theoretical structure (perfect in dimensions, composed of homogeneous materials having straight-line stress-strain relations, etc.), the stresses and strains at every point in it for any possible application of load can be determined. The hypotheses, that plane sections remain plane in bending, that E, the modulus of elasticity, is constant, assumptions of general application, and those used in special cases, as for dams, that horizontal sections move frictionless on each other, etc., by means of which calculation of the stresses and strains are made possible or greatly simplified, are, in most part, only close approximations, frequently giving results of the same magnitude. Their use, however, always gives magnitudes for the variables, that will not be exceeded at the specific point in the structure by the actual value of the variable.

Contrasting with the complete development of the methods of mathematical analysis of the stresses and strains in a structure are the crude methods used in the determination of the stresses which may be safely withstood by materials or composite structural elements, or the determination of the variations of the actual stresses from the theoretical for each point in the material or structural element composing the structure. As is usual with an undeveloped branch of investigation so in the determination of the working stresses to be allowed, extraneous factors are included as essentials in the consideration, no systematic procedure has been developed, and the whole subject is vague and indefinite, clouded with engineering superstition, such as the belief that repetition of stress (fatigue) causes crystallization in ferrous metals.

The present allowable working stresses are based primarily on experience. If the original stress was too small, the resulting failures, in the course of time, cause increase in the factor. It seldom happened that the stress was made too small, the common occurrence being an allowance much too large, which is slowly reduced to the point where minimum strength with safety is found. Undoubtedly, the experience method eventually produces good results; but it is rather dangerous and is exceedingly slow, tedious, and costly in operation. Without the testing machine, the experience, or trial-and-error, method is the only one possible. There is no longer an excuse for using this antiquated method. With the wealth of new materials, old materials made by new and refined processes, and as a check on the strength to be allowed those materials the strength of which constantly varies with the output, there is urgent need for a short method of determining definitely the allowable working stresses directly from the results of tests made in the common forms of testing machines.

2.—Definition of the Problem

The strength of the structure and the loads that it will carry is a function of the stresses in the members and the strength of the material or element of the structure in which these stresses are developed. The design of the safe structure necessitates the knowledge of the maximum stress that the material at any designated point or individual structural element can carry with safety, and, also, the knowledge of the maximum stress that the structure, due to all the variations and imperfections of the actual structure and its component materials and elements from the theoretical and perfect, together would impose on that material or individual structural element. The loading which would cause the maximum safe stress at any point in the structure, would then be the maximum permissible loading on the structure, and, as a corollary, the maximum permissible loading would cause, at most, the safe working stress at any point in the structure.

The determination of safe working stresses, therefore, does not involve details apart from the material or structural element the safe stress of which is desired. The safe stress of concrete in compression is independent of everything except the ability of that concrete to resist an infinite number of applications of stress of the magnitude to be designated the safe working stress.

The problem of ascertaining the maximum stress that can be developed in each particular member of a structure by the application of load, is one of design independent of the consideration of safe stresses, and should be so considered. The structure should be designed so that with any and all possible imperfections, mishaps, etc., the maximum permissible or designated loading on it will cause, at most, a stress equal to the safe working stress.

Increase in the factor of safety, a result of including too many and extraneous factors, does not give increased safety. Human nature is such that increase in the factor of safety leads to increased risks in loading and much heavier over-loading, with the result that the total imposed load will bear even a larger ratio to the ultimate strength of the structure for the larger factor of safety. The smaller the factor of safety the safer a structure becomes, as an overload of 100% for a factor of 2 is judged more accurately than an overload of 900% for a factor of 10. Consequently, all influences tending to increase the factor of safety should be eliminated, if possible.

3.—Previous Analyses of the Problem

Professor Talbot* contends that the working stresses and working factors in structures are influenced by:

1.—Lack of uniformity of material, variation in material and its fabrication.

2.—Increased deflection due to: (*a*) repeated load; and (*b*) time load.

3.—Uneven distribution in load among members.

4.—Unconsidered stresses due to settling, unevenness in material, etc.

* *Bulletin No. 10,* Univ. of Illinois, Eng. Experiment Station, Paragraph 24.

Professor Talbot also gives the following as important in the problem:

"(a) the stress actually brought upon a member of a structure by an assumed loading may be materially higher than the assumed working stress, and (b) the stress actually developed in the member may be much higher comparatively (i. e., with respect to its own ultimate strength) than even this increased amount would indicate. * * * What point should be fixed upon as the basic point, upon which a working factor covering uneven distribution of load, uncertainty of quality, effect of repeated loading, etc., may be based, will depend upon the nature of the material and the conditions of the structure. * * *"

And, in the summary,

"10.—* * * as a basic point [that is, a basic stress in the determination of the allowable working stress] * * * the choice of a value corresponding to a deformation equal to one-half of the deformation at point of failure is suggested. This [stress] by the parabolic relation, is equal to three-fourths of the ultimate strength. Having selected a basic point, a working factor will then be chosen to cover contingencies and emergencies, * * * methods of fabrication, nature of the load, manner of application, etc."

The whole problem may be censured because of indefiniteness and the inclusion of too many independent factors, many of which can be treated separately.

As a base stress on which to derive the working stress, the figure, three-fourths of the ultimate stress, can have no other basis than intuition. It has been shown in Section III that, to allow for the usual variation in the strength of concrete, one-half the ultimate strength must be used to insure safety. Professor Talbot leaves us about where we started when he says that this basic stress must be corrected for contingencies and emergencies, nature of load, etc. The problem has only been begun at this point. The usual method of solving it is to keep in mind all the factors influencing the problem, to meditate gravely the question and, then, announce with the conviction of infallible intuition, that the factor should be 4. This method may be convincing in metaphysical speculation, but it is much out of place in the solution of engineering problems.

Professor Talbot's items may be separated into:

A.—Those quantities which influence the stress actually produced in any particular member of the structure: Uneven distribution of load among members, unconsidered stresses due to settling, unevenness in the material, contingencies and emergencies unforeseen, methods of fabrication, etc.

B.—Those factors that influence the actual (safe) stress to be permitted in the material or structural element as in the structure. These factors are:

1.—Lack of uniformity of material, variation in material and in its fabrication.

2.—Increased deflection (preferably strain) due to: (*a*) repeated loads; and (*b*) time loads.

3.—Uncertainty of quality, effect of repeated and time loading, all of which solely influence the ultimate strength or safe stress which is to be imparted to the material or structural element.

Unevenness in material has been placed under both Items *A* and *B*. It must be considered twice, as it causes variation in the stress actually in a given member, and, also, in the ultimate strength of that member. Variation in strength of one member may cause a variation in stress in some other member.

As previously stated, the items referred to in Item *A* are problems of design and should not be investigated in consideration of the working stress. A unit volume of concrete has the same ultimate strength no matter where it is placed, and it should be permitted an invariable working stress. Its position in the structure does not change its safe stress.

The net result of the lack of uniformity and variation in material may be measured with precision by the reliability number, as demonstrated in Section III, and allowance made for the effect of the variation in strength. The variation in fabrication can be dealt with in two ways, each of which will give decisive results. The first and best method is to take test specimens on the work as a check on the workmanship and leave no variable for this quantity. There will be, therefore, no need for introducing a factor into the allowable working stresses to provide for the variation. The second method is to determine the ratio of the strength of the specimens made in the laboratory to those made in the field. This ratio will then give the field strength of a certain aggregate, if laboratory specimens are made from it. An excellent series of such tests is described subsequently, which give valuable results. It can now be seen that the "uncertainty of quality" and "variation in material and in its fabrication" can be eliminated.

The three items of uncertainty and variation mentioned have, in effect, only one net result, namely, the variation in strength of the material in the structure. This may be due to uncertainty of material, an ambiguous term, to variation in strength due to fabrication, and in differences in erection, such as the treatment during the preparation of the concrete, or the erection of the structural steel. The variation in strength due to fabrication can be eliminated, as the material to be used can be tested and its strength and other properties determined before it is incorporated in the finished structure. In design, a certain fabricated strength is specified, and the material as delivered to the work must possess this strength as a minimum. The variations in strength of the material in the finished structure—the most important item—has been discussed in Section III and will be reviewed subsequently. The increased deflections (more properly, the increased strains), and the decreased ultimate strength due to repeated and time loading, are of vital importance.

Messrs. Turneaure and Maurer* give the following items governing the selection of the allowable working stresses:

(*a*) Variation and imperfection in material and workmanship.

(*b*) Uncalculated stresses, such as secondary stresses due to unequal settlement, and, usually, those due to temperature change.

* "Principles of Reinforced Concrete Construction," Second Edition, Chapter V, p. 209. No mention of the subject is made in subsequent editions, the authors evidently leaving the matter in the hands of committees of the proper engineering society.

(*c*) Dynamic effect of live load if not provided for by an allowance for impact.

(*d*) Possible increase in live load over that assumed, or rare applications of excessive loads.

(*e*) Deterioration of the structure.

These and similar items are important only in proportion to their application. No further use of them than their enumeration makes this mention superfluous. In this instance, the statement of the items seems to have been sufficient, for later in the book, without any further reference or application of them in the intervening pages, the allowable working stresses for a concrete column reinforced with vertical rods is determined by "taking" an ultimate stress that can "reasonably be expected" for such a column, and dividing this by an arbitrarily chosen "factor of safety of 4". The use of the phrase, "reasonably be expected", and the arbitrary selection of 4 as the factor of safety are hardly in keeping with a branch of knowledge with such a scientific basis as Engineering. These methods are not based on any processes that can be logically followed by different individuals to obtain identical results. Had some other authority given the factor, it might have been different. It is true that the factor of safety for reinforced concrete is taken by the best engineers and, in general, accepted as about 4. The Joint Committee on Concrete and Reinforced Concrete in its 1917 Report, suggests 22½ per cent. The difference in using a factor of 3 instead of 4 results in a saving of 25% of material. Concensus of opinion would have it that a factor of 3 is too low; yet how, by this method of intuition, can it be disproved that 4 is too high and that 5 is the lowest factor to be used? Can this vital matter be decided by the opinion of the majority? Failures in concrete structures are of too frequent occurrence. On this basis alone, the factor of safety should be increased; yet, it is well known that all these failures are due to inferior concrete, and instead of increasing the factor of safety, a rigid control of the making and placing of concrete and an insistence on a high strength should be strongly urged.

Item (*a*), variation and imperfection in material and workmanship, is a factor vitally affecting the allowable working stress. Although it has long been recognized that there are variations in the strength of materials and structural elements, especially pronounced in concrete, due to "variation and imperfection in material and workmanship", no method has been used to determine even crudely the affect of such variation and imperfection on the strength. The influence of imperfection and distortion of the structure, such as that indicated in Item (*b*), is a consideration of design and in no way affects the working stress. There must be some limit to the factors that must be recognized and met in the figure set for the safe working stress. To illustrate the reason for this necessary limitation by an exaggerated case, no factor of safety could ever compensate for the omission of all the cement from a batch of concrete. It is possible to determine with reasonable accuracy the stress that a material or structural element will safely carry. If part of the problem is capable of solution, why introduce unnecessary, unknown, and problematical factors such as emergencies, possible overloads,

unequal loading, eccentric loading, settlement, etc. These factors vary with the individual structure, the foundation, and the use to which the structure is placed. The emergencies and overloads should be the utmost conceivable for the particular case. The design is the proper place to take cognizance of all these factors. The problem of settlement should be solved by the use of foundation areas proportional to the load on them, and the structure so designed, possibly by a change from the present monolithic construction, in order that unequal settlement will not cause secondary stresses or unequal distribution of load directly or indirectly due to it.

The dynamic effect, as in Item (*c*), should always be considered. Working stresses cannot be determined from two independent criteria. There is no way to allow for a rare application of an excessive load, except to design so that such a load produces, as a maximum, the working stresses in the individual members. The working load would then be a percentage of this rare excessive load. The phenomenon of fatigue reduction of strength permits a few applications of 100% overload, but this is an inherent property of materials, and cannot be adjusted.

Item (*e*), deterioration of the structure, is ambiguous. It can be taken to mean reduction in the ultimate strength due to time loading, to loss of strength in time independent of the load, or to reduction in strength due to loss in area of the structural elements by abrasion, disintegration, or other influences.

It has been determined experimentally that concrete does not lose any material part of its maximum strength with age. The effect of time loading has been determined definitely. For the particular case, the quantity of material lost by abrasion during the life of the structure must be predetermined and extra material added to the structural element. Disintegration should be prevented, or if that is impossible, allowance for it should be made. Such effects as abrasion, disintegration, etc., cannot be considered, being factors of design and unrelated to determinations of safe stress.

4.—A Rational Scheme for the Determination of Safe Working Stresses

It is not necessary to consider internal column reactions of the nature of the initial strain and correlated stress on reinforcing rods in longitudinally reinforced concrete columns due to shrinkage of the concrete, as these phenomena occurred in the test columns. The results showing a stress of 31 000 lb. per sq. in. in the reinforcing rods of longitudinally reinforced columns possibly indicate that initial compression due to concrete shrinkage was present, as claimed by several authorities. Neither is it necessary to take into consideration the method of failure of the column or compound structural element. It is immaterial, for example, whether the rods of a longitudinally reinforced concrete column cause the failure of the column or whether the failure is independent of the action of the rods. The safe working stresses can be based on formulas determined mathematically from the results of column tests, eliminating all speculation on the internal behavior of the column and its elements.

The safe working stresses for a type of structural element must be a function of the following factors:

1.—The maximum stress that can be applied in infinite repetition, or for one sustained application for an infinite period (time load), without causing (*a*) a permanent set, or, (*b*) a strain at the last repetition or end of the infinite time load, either being greater than some maximum permissible strain value determined by factors inherent in the structure itself or otherwise determined, such as, by the presence of plastered coating on the walls. For an elevated railway structure, for example, a working strain of 0.0035 in. per in. in the supporting members would not be excessive.

2.—The formula, empirical or exact, that has been confirmed experimentally, expressing in terms of all the component variables the strength of the structural element.

3.—The reliability number, N_r, based on the variation in strength of the individual specimens from the strength indicated by the formula given in Factor (2).

4.—The relation between the strengths of the specimens prepared in the laboratory (such as are always used in tests) and the strength capable of being developed by the material as commercially fabricated. When check test specimens of the concrete as mixed on the work are taken, this factor is unnecessary.

The safe working stress, or preferably the safe working load, reduces to a simple equation of the type:

$$P_m = N_r \, L_e \, F\,(V \;\; M f_c)\, A \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

in which

P_m = the safe column load;
N_r = the reliability number;
L_e = the endurance limit or maximum stress possible on account of strain or set consideration;
$F\,(V\,.\,M f_c)$ = the mean strength of the column empirically determined; a function of the variables: (*a*) strength and percentage concrete; (*b*) percentage steel reinforcement; and (*c*) slenderness ratio, etc.
f'_c = the ultimate stress of concrete test cylinders;
M = a factor equal to unity when check test specimens are taken of the concrete, and, approximately, 0.6 when they are not;
A = the cross-sectional (active) area of the column (equal to the area within the spiral diameter in that type of column).

A.—Essential Data on Concrete

Fatigue.—J. L. Van Ornum, M. Am. Soc. C. E., has found,* by means of large numbers of compression test cylinders and also test beams reinforced so as to fail in compression in the concrete, that the endurance limit for concrete is 50% of its ultimate strength, proper methods being used to eliminate the effect of variation in strength in the concrete of the individual test specimens. The curve illustrating the fall in strength with the number of repetitions is given in Fig. 13.

* *Transactions*, Am. Soc. C. E., Vol. LI (1903), p. 443, and Vol. LVIII (1907), p. 294.

The strains resulting from repeated applications of load must also be obtained. Professor Van Ornum has done this, obtaining the modulus of elasticity, E_c, for each additional application of stress throughout the cycle. He found for stresses of more than 50% of the ultimate stress, that each application of load decreased E_c, the greater the stress ratio to the ultimate stress, the more rapid the decrease, with the final result that the concrete failed in compression. He also found for the endurance limit stress (50% of the ultimate), or greater, that E_c decreases, but reaches an asymptotic value. For an initial value of $E_c = 3\,500\,000$, and an application of the endurance limit stress, the asymptotic value is $E_c = 2\,250\,000$, 65% of the initial value. For this loading, the permanent set is quickly achieved and does not increase. This property of concrete is of great value. From stress considerations, the endurance limit is the maximum stress value that can be used; from purely strain considerations, the endurance limit may also be used.

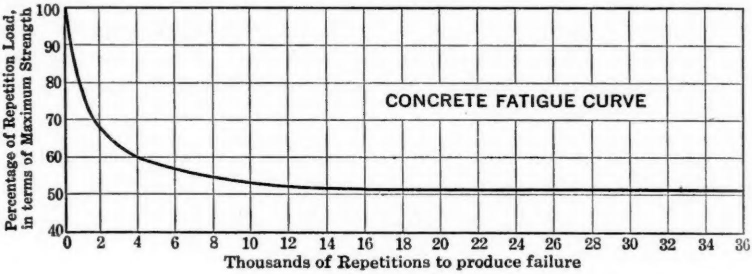

FIG. 13.

B.—The Relative Strength of Laboratory Made and Commercially Made Concrete

The correct methods of making and preserving concrete to develop maximum strength, prevent deterioration, etc., have been developed and perfected by laboratory research. Commercial concrete cannot be expected to be made with all the necessary refinements and precautions, because of the high inherent cost and the impossibility of placing the strongest concrete properly in complicated reinforcement. There is no reason, however, to tolerate the present general product that is furnished.

The results of a series of tests given in *Technological Paper No. 58,* of the U. S. Bureau of Standards, will be used as data in this discussion of working stresses. The results are given of tests on the strength developed by a definitely proportioned dry aggregate as mixed and made into concrete test cylinders by several contractors and on similar specimens made by the Bureau. The data are not exactly those desired in this instance, as the contractors wanted to make the strongest possible concrete instead of the usual run, the cylindrical test specimens permitting them to make a stronger concrete, the aggregate and proportions being correctly chosen. If left to the contractors, the resulting aggregate would cause a further reduction in strength. The results of these tests are given in full in Table 11. It will be seen that the average strength developed by the Bureau was $66\frac{2}{3}$% higher than that developed

by the contractors. This difference was due to variation in thoroughness in mixing, placing, and, chiefly, the too great content of water. In the fabrication of difficult reinforced shapes, it would be almost impossible to use the quantity of water necessary to develop maximum strength for a given aggregate. Aside from the special conditions where it is impossible to use the quantity of water to develop the maximum strength, flagrant abuses such as a large surplus of water, insufficient mixing, incorrect proportions, incomplete tamping, etc., will not tend to furnish a concrete as strong as that produced in the laboratory. It would be insufficient to have all technical men thoroughly familiar with the principles of making concrete. The average workman will do his work in the easiest possible manner, and although he will do it to conform with the specifications in the presence of the inspecting engineer, he will lapse in his absence. The only positive check on the quality of the concrete is the test cylinder.

TABLE 11.

Age at test, in weeks.	Laboratory.	Contractor.			Laboratory.	Contractor.			Proportions.	Mixture and aggregate.
		A	B	C		A	B	C		
	Ultimate stress, in pounds per square inch.				Percentage of laboratory strengths developed.					
4	700	880	450	580	100	125.6	64.3	82.8	1:3:6	Hand and machine mixed, gravel.
	.....	610	560	400	....	.....	....	....	....	....
	1 010	850	560	670	100	84.0	55.3	66.3	1:3:6	Machine mixed, gravel.
	950	420	400	690	100	44.2	42.2	72.7	1:3:6	Machine mixed, limestone.
	3 050	1 250	1 620	2 210	100	35.8	46.3	63.2	1:2:4	Hand and machine-mixed, gravel.
	2 310	1 450	1 790	1 820	100	62.7	77.5	78.8	1:2:4	Hand and machine-mixed gravel.
Mean.	.....	.....	.....	.....	100	70.5	57.1	72.7	66.8	Mean of three means.
13	1 100	880	590	690	100	80.0	53.7	62.8	....	....
	1 610	1 260	950	1 120	100	78.2	59.0	69.5	....	....
	1 210	740	550	850	100	61.2	45.5	70.3	....	....
	3 150	1 750	2 160	2 120	100	55.6	68.6	70.5	....	....
	3 320	1 900	2 980	2 830	100	57.3	90.0	85.5	....	....
Mean.	.....	.....	.....	.....	100	50.5	52.6	59.2	54.1	Mean of three means.

5.—Working Stresses

An abstract of the working stress specifications for reinforced concrete columns for the principal American cities, and calculations of the stresses based on them for all large cities in the United States, has been published by Mr. Pierce P. Furber.*

The Specifications recommended in 1917 and the Tentative Specifications submitted in 1921, by the Joint Committees are to be accepted as more nearly

* "A Summary of the Column Code", *Journal*, Am. Concrete, Inst., 1916, p. 181.

the standard to which in time the various codes will be revised, and have been chosen as the basis of the general criticism.

The 1917 Specifications* for "Columns" are, as follows:

"(*a*) Columns with longitudinal reinforcement to the extent of not less than 1% and not more than 4%, and with lateral ties of not less than ¼ in. in diameter, 12 in. apart, nor more than 16 diameters of the longitudinal bar: the unit stress recommended for axial compression, on concrete piers having a length of not more than four diameters, in Chapter VIII, Section 3. [This stress for the concrete piers is 22½% of the expected strength of the concrete.]

"(*b*) Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with circular hoops or spirals not less than 1% of the volume of the concrete and as hereinafter specified; a unit stress 55% higher than given for (*a*), provided the ratio of unsupported length of column to diameter of the hooped core is not more than 10."

Chapter II of the 1917 Specifications† on "Design and Supervision", contains the following:

"(*d*) Inspection * * *:

"5. Strength of the concrete [to be obtained] by tests of standard test pieces made on the work."

The 1921 Tentative Specifications‡ are, in substance, as follows: The safe load on spiral reinforced columns is given by the equation:

$$P = A_c f_c - n f_c p A \qquad (A)$$

in which,

A = the area of the concrete core enclosed within the spiral;

P = the total safe axial load on a column whose $\frac{h}{R}$ is less than 40 (R is the radius of gyration of the cross-section of the column within the spiral, and h is the column height);

p = the ratio of effective area of longitudinal reinforcement to area of concrete core;

$A_c = A\,(1 - p)$ = the net area of concrete core;

$n = \frac{E_s}{E_c}$ (for which the load is not stated);

f_c = the safe load on the unit of core cross-section and is given by the equation:

$$f_c = 300 - (0.10 - 4\,p)\,f'_c$$

f'_c = the crushing strength of concrete in 6 by 12-in., or 8 by 16-in., test cylinders.

"The longitudinal reinforcement shall consist of at least six bars of minimum diameter of ½ in., and its effective cross-sectional area shall not be less than 1% nor more than 5% of the enclosed core. * * * The spiral reinforcement shall not be less in amount than one-fourth the volume of the longitudinal reinforcement. * * * Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core, which shall have a minimum thickness of 1½ in. in square columns, and 2 in. in round or octagonal columns."

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* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1133.

† *Loc. cit.*, p. 1111.

‡ *Proceedings*, Am. Soc. C. E., August, 1921, p. 102.

Section 67* of the 1921 Specifications states that the reinforcing shall be "not less than 2 in. in beams, girders and columns" where the aggregate is limestone, or its fire-resisting equivalent, or 3 in. for quartz.

The safe load for longitudinally reinforced columns is given by the equation:

$$P = A'_c f_c - A_s n f_c \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (B)$$

in which,

$A'_c =$ the net area of concrete in the column (total column area less steel area);

$A_s =$ the effective cross-sectional area of the longitudinal reinforcement;

$f_c = < 0.20 f'_c$, and is the safe load on the unit of column cross-section;

$f'_c =$ the crushing strength of the concrete in 6 by 12-in., or 8 by 16-in., test cylinders.

The longitudinal reinforcement must not be greater than 2% nor less than 0.5 per cent.

For both types of columns with $\frac{h}{R}$ larger than 40, the safe load is given by the equation:

$$P' = P\left(1.33 - \frac{h}{120\,R}\right)\dagger \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (C)$$

in which, P corresponds to values given by Equations (A) and (B) and P' is the value of the safe load for the column of the given value of h and R.

In Table 4,‡ Section 2 of the 1921 Specifications, it is stated that:

"If the proportions to be used in the work are selected from the table without preliminary tests of the materials, and control tests are not made during the work, the mixtures in bold-face type shall be used."

A.—Columns Reinforced with Vertical Rods Only

The analysis in Section I shows that:

1.—The rods at ultimate column strength are stressed approximately to their yield point.

2.—The yield-point strength in the reinforcing rods was developed independently of the amount of the reinforcement, a few of the test columns having as much as 4% of rods. The strength developed by the larger percentage is capable of much closer check because of the far greater influence on the column strength than a relatively small percentage, such as 1 per cent.

3.—The rods do not decrease the strength developed by the concrete in any individual column, and the concrete does not decrease the strength of the rods. The contention is untrue that, in the process of destruction of the column, the rods buckle and cause the surrounding outer layer of concrete to scale, thus starting and aiding the failure of the column. The phenomenon is

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* *Proceedings*, Am. Soc. C. E., August, 1921, p. 80.

† Equivalent to $P' = P\left(1.33 - 0.033\,\frac{L}{D}\right)$.

‡ *Proceedings*, Am. Soc. C. E., August, 1921, p. 111.

a phase in the failure of the testing of this type of column, and represents the buckling of the rods after the maximum strength of the column and concrete has been passed and the concrete has broken, allowing the rods to buckle outward.

4.—The strength of the column varies considerably within the limits of the lengths of columns tested and permitted in practice.

5.—To develop the full strength in the reinforcing rods, it is necessary that the ultimate strain of the concrete reinforced shall be, as a minimum, the strain necessary to develop the full yield-point stress in the steel reinforcement, which is 0.00125 in. per in. The minimum ultimate stress of concrete to develop this minimum ultimate strain is 2 000 lb. per sq. in.

6.—The reliability of the strength of the rods and of the concrete developed in the rod reinforced columns is independent of the amount of the rod reinforcement.

Besides these five factors governing the safe working stresses to be permitted and of which no cognizance is taken in the present and tentative code, there is another important factor, namely, no credit as regards strength is given the protective shell of the spirally reinforced column, yet there is no concomitant reduction made from the actual concrete cross-sectional area of the longitudinally reinforced column.

If the spirally reinforced column requires fire protection, the rod reinforced column does also. Fire will destroy the surface concrete of the rod reinforced column and decrease the strength-bearing area in the same manner as in the spirally reinforced column; and, as is proper, the strength developed by the shell of the spirally reinforced column is not considered. A shell of equal thickness should be used as fire protection in the rod reinforced column, and no strength credit given to this layer of concrete. That the shell of the spirally reinforced column is destroyed at ultimate column strength is a valid reason for not allowing credit for it in calculations of safe working stress. Heretofore, however, the ultimate strength of the spirally reinforced column has never been the basis of the calculations for working stress as the ultimate strain is so large, the ultimate strain of unreinforced concrete being the basis for the determination of the allowable working stresses. The protective shell adds strength to the column at the ultimate strain of unreinforced concrete, exactly as the outer layer of concrete adds strength to the rod reinforced column. If this layer is given no credit in the spirally reinforced column, due to its function as fire protection, then certainly a layer of equal thickness should be allowed for fire protection in the rod reinforced column, and given no strength credit.

Development of the Formula.—The formula giving the column strength relative to the necessary variables, as developed in Section I, is:

$$\frac{P_m}{A} = 1.28\left(r_c\, f_c + 31\,000\, r_r\right)\left(1.00 - 0.0265\,\frac{L}{D}\right) \ldots\ldots\ldots (55)$$

It is now necessary to correct Equation (55) by the reliability number, N_r, to allow for the natural variation in strength, and by the factor, L_e (= 0.5),

to allow for repeated and time loads, that is, to obtain the endurance limit. Equation (55), corrected by these factors, then becomes:

$$\frac{P}{A} = 1.28\ L_e\ N_r\left(r_c\ f'_c + 31\ 000\ r_r\right)\left(1.00 - 0.0265\ \frac{L}{D}\right) \ldots\ldots (56)$$

in which,

P = the safe column load;
A = the effective column area (that is, the column area less the outer layer of concrete designated as fire protection);
r_c = the ratio of concrete in the column $(1.00 - r_r)$;
r_r = the ratio of longitudinal reinforcing rods;
f'_c = the average ultimate strength of the concrete as used in the column and determined by test cylinders;
L = the length of the column;
D = the diameter of the effective concrete area in the same units as the length of the column;
N_r = the reliability number; and
L_e = the endurance limit for concrete (0.5).

For design, N_r is either the value that may reasonably be expected from a given aggregate and proportions, or that may be determined by test specimens. In the actual structure, $N_r\,f_c$, as determined by test specimens, must be, as a minimum, the value assumed for design.

For the rod reinforced column, analyzed in Section I, $N_r = 0.57$, the figure used in the safe stress computation in this Section.

Equation (56) now becomes:

$$\frac{P}{A} = 0.365\left(1.00 - 0.0265\ \frac{L}{D}\right)\left(r_c\ f'_c + 31\ 000\ r_r\right) \ldots\ldots\ldots\ldots (57)$$

An alternate procedure, not to be recommended, is to be used when no check test specimens are taken of the concrete as mixed on the job. Although haphazard, this method is possibly the only plausible one in small work and in cases when those in charge cannot be convinced of the necessity of test specimens. The alternate method of determining the permissible stresses is to incorporate a factor of 0.6 in Equation (57) to correct for the strength of the concrete as determined by test specimens made from laboratory mixture.

Equation (57) would then read:

$$\frac{P}{A} = 0.365\left(1.00 - 0.0265\ \frac{L}{D}\right)\left(0.6\ r_c\ f'_c + 31\ 000\ r_r\right) \ldots\ldots\ldots (58)$$

The constant, 0.6, was obtained from *Technological Paper No. 58* of the U. S. Bureau of Standards, and is the ratio of the strength of concrete mixture by skilled laboratory workers to that obtained by employees of contractors. Although the concrete as mixed on the work would not in every case be only 60% of the strength of concrete mixed in the laboratory, yet, if no test specimens are taken, the capable makers of concrete must be penalized equally with the makers of ordinary concrete.

Table 12 has been compiled to demonstrate the great variation in the safe working stresses for columns of extreme dimensions, and also the great

difference between those computed on the strength of concrete, as usually obtained, and the present allowable working stresses.

TABLE 12.—SAFE WORKING STRESS TABULATION FOR ROD REINFORCED COLUMNS.

A	B	C		D	E	F	G	H
Concrete, ultimate strength, in pounds per square inch.	Ratio of length to diameter, $\frac{L}{D}$.	PERCENTAGE OF REINFORCEMENT.		PERMISSIBLE WORKING STRESSES, IN POUNDS PER SQUARE INCH BY		CALCULATED SAFE STRESSES, IN POUNDS PER SQUARE INCH BY		Working stress in concrete, in pounds per square inch, for stress indicated in Column F.**
		Spiral.	Rods.	1917 Specifications.	1921 Specifications.	Equation (57).	Equation (58)¶.	
			0.50†	....	321	505	322	
			1.00‡	338	342	552	370	
1 500	6	0	2.00§	338	384	642	461	425
			4.00‖	338	....	823	647	
			10.00	....	....	1 365	1 200	
			0.50	....	300	410	262	
			1.00	338	319	447	300	
1 500	12*	0	2.00	338	359	520	375	425
			4.00	338	....	666	525	
			10.00	....	....	1 110	975	
			0.50	....	268	362	230	
			1.00	....	285	395	264	
1 500	15	0	2.00	....	321	460	330	425
			4.00	....	....	588	462	
			10.00	....	....	975	855	
			0.50	....	633	960	597	
			1.00	675	666	1 006	642	
3 000	6	0	2.00	675	732	1 092	730	850
			4.00	675	....	1 265	911	
			10.00	....	....	1 780	1 450	
			0.50	....	597	747	465	
			1.00	675	630	783	498	
3 000	12*	0	2.00	675	692	850	570	850
			4.00	675	....	984	710	
			10.00	....	....	1 382	1 125	
			0.50	....	528	686	426	
			1.00	....	556	720	458	
3 000	15	0	2.00	....	611	780	522	850
			4.00	....	....	905	651	
			10.00	....	....	1 270	1 035	

* Maximum length permitted by 1917 Specifications.
† Minimum permissible under 1921 Specifications.
‡ Minimum permissible under 1917 Specifications.
§ Maximum permissible under 1921 Specifications.
‖ Maximum permissible under 1917 Specifications.
¶ Based on actual concrete of 60% of the strength of laboratory specimens.
** Actual stress in the concrete of the column under working load. Column F gives average stresses in both steel and concrete.

Criticism of the 1917 Specifications.—

1.—The allowable working stresses bear very little relation to the safe stresses, as determined by the writer.

2.—Based on the strength of concrete to be expected from a mix of a certain nominal strength, the working stress permitted varies as much as 40% greater than the safe stress as determined by the writer.

3.—If concrete is well made and check specimens are taken, much higher (as much as 100%) safe stresses can be sustained than are permitted by the 1917 Specifications.

4.—The permissible working stress should vary with the percentage of reinforcement and the length of the column.

5.—The maximum percentage of permissible reinforcement is not large enough. Tests show that as much as 4% of reinforcement develops the same stress *per se* as the least percentage of reinforcement.

6.—A thickness of shell equal to that demanded for spirally reinforced columns as fire protection should be required in longitudinally reinforced columns, no strength allowance being given to the shell.

Criticism of 1921 Tentative Specifications.—

1.—The allowable stresses disagree with the safe calculated stresses, although not as widely as those in the 1917 Specifications.

2.—The 1921 Tentative Specifications are an improvement on the 1917 Specifications in that they (*a*) permit longer columns, and (*b*) varying working stress, within the limits.

3.—The 1921 Tentative Specifications have less merit than the 1917 Specifications in that they decrease the already too small maximum permissible percentage of longitudinal reinforcing rods.

4.—The working stresses, in all cases, are lower than the safe calculated stresses provided the concrete in the column is equal to the nominal concrete strength on which design has been based, and for many choices of mix and percentage reinforcement are lower than the safe calculated stress by as much as 25 per cent.

5.—For the strengths to be expected from a nominal strength concrete, the permissible stresses of the 1921 Tentative Specifications are much above the safe calculated stresses for many possible selections of variables.

Comment on Stress Values Determined in This Paper.—

1.—The unit working stress in the concrete of the column as determined in this discussion is not excessive, being only 28.4% of the ultimate strength.

2.—With this conservative stress from the old general viewpoint, much greater total column loads are obtained.

3.—The great increase in strength due to increase in reinforcement is demonstrated.

B.—Columns Reinforced with Longitudinal Rods and Spiral

The analyses in Sections I and II show that:

1.—The increase in strength of concrete due to spiral reinforcement is proportional to the strength of the concrete and to the percentage of spiral reinforcement to a maximum of 2%, there being no experimental data for a larger percentage of reinforcement.

2.—The increase in strength of the concrete due to the spiral reinforcement is independent of the ultimate strength of the spiral when this value lies between the limits of 60 000 and 150 000 lb. per sq. in.

3.—Longitudinal reinforcing rods in a spirally reinforced column contribute approximately 36 500 lb. per sq. in., or the yield-point stress per unit area of rod, to the column strength at maximum load, the data being available only for a maximum rod reinforcement of 10 per cent.

4.—To the ultimate strain of approximately 0.0015 in. per in., the protective shell is undestroyed and in great part is intact and serviceable as a fire protection at the ultimate column strain (maximum column load). At the strain of 0.0015 in. per in., the protective shell considerably increases the load supported by the column depending on the ratio of the area of the shell to that of the core. This increase is approximately 50%, and depends on the ratio of core to shell areas.

5.—The column strain at maximum load, or ultimate column strain, is approximately 0.0035 in. per in.

6.—The reliability of the ultimate or maximum strength of this type of column is only slightly less than for unreinforced concrete or for concrete reinforced with longitudinal rods.

7.—The strength reliability does not vary with: (*a*) the amount of spiral reinforcement with a maximum of 2%; (*b*) the amount of longitudinal rod reinforcement with a maximum of 10%; and (*c*) the column length with twelve column diameters as a maximum.

From an inspection of these qualities possessed by the spiral and rod reinforced column, it is at once apparent that the safe working stresses permitted should depend on the percentage of reinforcement, the length, and the relation of the other column variables. This is more clearly brought out in Table 12, of the calculated safe working stresses, which are shown to vary almost 100% for the limits of the variables permitted by the 1917 Specifications for the one permitted working stress.

Continued compression of specimens of the spiral and rod reinforced type of column, far beyond the ultimate and maximum stress and strain, has caused the general destruction of the shell, the spirals to be broken by tension, buckled longitudinal reinforcing rods, buckling of the column itself, and, in general, a complete destruction of the column proper. This has led to a general belief that these phenomena occur simultaneously with the point of maximum load, and a refusal to accept the ultimate column strength as the basis for the calculations of the safe working stresses. It is immaterial how a structural element fails. When failure occurs, the element is useless and, therefore, all other elements supported by it or transmitting loads to or from it are likewise useless in the system.

The ultimate column stress should be the basis of the calculations of the safe working stress. That some procedure has been adopted at some time or place is with many of far greater weight than the most logical theory, no matter how positive and incontrovertible the proof. Refer to the working stresses suggested by Considère and adopted by the French Commission, and given for the rod and spiral reinforced column by the formula:

$$f = f_c\ (1 + 15\ r_r + 32\ r_s)$$

in which,

f = the safe working stress;

f_c = the safe strength of the plain concrete, taken at 28% of the ultimate strength of the concrete in the form of cubes (equivalent to $18\frac{1}{2}$% of the strength in the form of cylinders, according to the writer);

$r_r =$ the ratio of longitudinal reinforcement;

$r_s =$ the ratio of spiral reinforcement, with the criterion that the maximum working stress permitted shall be 0.6 the ultimate strength of the concrete.

In passing, it may be noted that the maximum stress permitted by Considère is about 73% greater than that permitted by the 1917 Specifications of the Joint Committee.

Returning to the development of the logical argument for the ultimate stress as the basis for the determination of the safe working stress: The arguments against such a determination are:

1.—The unreliability of the strength developed by the concrete.

2.—The column buckles.

3.—The column reaches maximum (ultimate) load at a very great strain, too great to permit in a structure.

4.—The marked yield point of the column.

These arguments may be answered in turn, as follows:

1.—From the results of the mathematical investigation in Section I, as previously summarized, the rod and spiral reinforced column is shown to be almost as reliable as the rodded column; the reinforcing elements contribute a definite part of the strength; and, finally, a mathematical equation being derivable, the ultimate strength reduced by the reliability number can be used accurately to determine the safe strength of the column, no matter how great the variations in strength may be.

2.—The column buckles after maximum load has been reached and passed. The steel column, generally recognized as perfection in structural work, also buckles after maximum load has been attained. The bend in the column in maximum load can be understood to be negligible from the consideration of the ultimate strain (that is, strain at maximum load). This strain at a maximum is 0.006 in. per in. Any bend in the column would manifest itself as an addition to this measure of compressive strain. Assuming 0.006 as due to the flexure of the column, a figure much too large, as the greater part of the measured compression is a compression strain in the column, the mid-point of a 10-ft. column would be laterally deflected about 1½ in. This amount of deflection or bending is not a condition to be permitted permanently, but is not excessive at the ultimate strength. The working stresses are far below this stress, and this condition of bending and concomitant ultimate stress is, by the process of derivation, never for safety to be equalled.

3.—The great ultimate strain of spirally reinforced concrete is an advantage instead of a disadvantage. It permits of distribution of the load throughout the structure. The great ultimate strain is also an advantage, because of the concomitant great ultimate resilience. The column can absorb, just previous to failure, a large amount of energy in comparison to the rod reinforced column.

4.—The fallacy of the yield point of spirally reinforced columns has been brought out in Section II and may be summarized, as follows: The initial column consists of core and shell, integral and intact. With increase in strain and stress, at the ultimate strain of concrete, the shell is gradually destroyed,

cracks, and falls from the column. The result is that the stress, previously distributed over the whole cross-section, is finally distributed over the core area, resulting, for a small increase in load, in a great increase in stress and a natural relatively sudden increase in strain. Thus, the yield point really is that point at which greatly additional stress is impressed on the core without being visibly denoted on the testing machine.

There is one more important argument for the ultimate stress as a basis, that is, the advantage of the protective shell, which may be summarized, as follows:

1.—The shell, commencing to crack at the ultimate strain of unreinforced concrete, gives warning of overload and also of possible failure. Reference to Table 13 shows that the actual safe working stress in the concrete itself is about 32%, as computed by the method developed in this study. Overload sufficient to cause noticeable cracking would then indicate 200%, certainly a large leeway in load variation and also a sufficient warning of impending failure, under about 100% still greater load.

2.—The shell is sound and assumes about 50% of the column load for the ordinary column proportions for all stresses within the working range.

3.—The shell will be decreased in strength in part or in whole, according to the destroying influences of fire or overload beyond the unit ultimate stress of unreinforced concrete. The shell, therefore, is sound and a perfect fire protection at normal working loads and stresses, which will not be the case in the event of both these influences, but the probability of the two occurring simultaneously is extremely small. If the fire protection is perfect there will be no overload in case of fire. Imperfect protection would possibly result in the collapse of upper floors, which would result in unequal distribution of the weight of such floors and the impact overload due to their fall.

Formula for Safe Working Stress.—Using the maximum or ultimate strength of the spiral and rod reinforced concrete column as the basic strength, the formula for the safe working stress in this type of column, derived as for the rod reinforced column, is:

$$\frac{P_m}{A} = 1.20\, N_r\, L_e \left[(1 - r_r)(1 + 0.573\, r_s)\, f'_c + 36\,500\, r_r\right] \times \left[1.00 - 0.0183\, \frac{L}{D}\right] \qquad (59)$$

in which,

P_m = the total safe load on the column;
A = the area of column cross-section, within the spiral;
r_r = the ratio of longitudinal reinforcing rods;
r_s = the ratio of spiral reinforcement;
f'_c = the average strength of the concrete, as tested in the form of small test cylinders;
L_e = the endurance limit, taken as 50% of the ultimate strength;
$\frac{L}{D}$ = the ratio of column length to diameter (of spiral); and
N_r = 0.46, as determined for the spiral and rod reinforced column in this paper.

TABLE 13.—WORKING STRESSES FOR SPIRAL AND ROD REINFORCED COLUMNS.

A	B	C		D	E	F	G	H	I	J
Concrete, ultimate strength, in pounds per square inch.	Ratio of column, core length to core diameter.	PERCENTAGE OF REINFORCEMENT.		PERMISSIBLE WORKING STRESSES, IN POUNDS PER SQUARE INCH.		CALCULATED SAFE STRESSES, IN POUNDS PER SQUARE INCH.		WORKING STRESS IN CONCRETE IN POUNDS PER SQUARE INCH, FOR STRESS INDICATED IN COLUMN F.		Permissible stress, in pounds per square inch,† spiral neglected, column of same dimensions as chosen in Column I.
		Rods.	Spiral.	1917 Specifications.	1921 Specifications.	Equation (59)	Equation (60)	Shell neglected.	Shell included.*	
1 500	6	1.00	0.25	......	582	506	340	395	253	510
		5.00	1.25	......	1 274	1 040	807	592	380	
		1.00	1.00	523	582	662	433	543	348	
		4.00	2.00	523	......	1 115	812	740	474	
		10.00	2.00	......	......	1 605	1 320	740	474	
1 500	10	1.00	0.25	......	582	466	311	395	253	510
		5.00	1.25	......	1 274	955	740	592	380	
		1.00	1.00	523	582	608	397	543	348	
		4.00	2.00	523	......	1 020	745	740	474	
		10.00	2.00	......	......	1 473	1 212	740	474	
1 500	15	1.00	0.25	......	486	413	277	395	253	425
		5.00	1.25	......	1 065	850	659	592	380	
		1.00	1.00	......	486	540	353	543	348	
		4.00	2.00	......	......	912	663	740	474	
		10.00	2.00	......	......	1 310	1 078	740	474	
3 000	6	1.00	0.25	......	800	923	590	790	506	1 000
		5.00	1.25	......	1 248	1 650	1 167	1 184	760	
		1.00	1.00	1 026	800	1 232	775	1 080	696	
		4.00	2.00	1 026	1 167	1 870	1 268	1 480	950	
		10.00	2.00	......	......	2 315	1 750	1 480	950	
3 000	10	1.00	0.25	......	800	848	541	790	506	1 000
		5.00	1.25	......	1 248	1 510	718	1 184	760	
		1.00	1.00	1 026	800	1 130	1 072	1 080	696	
		4.00	2.00	1 026	1 167	1 715	1 162	1 480	950	
		10.00	2.00	......	......	2 120	1 602	1 480	950	
3 000	15	1.00	0.25	......	668	753	481	790	506	853
		5.00	1.25	......	1 073	1 345	954	1 184	760	
		1.00	1.00	......	668	1 006	635	1 080	696	
		4.00	2.00	......	974	1 529	1 035	1 480	950	
		10.00	2.00	......	......	1 890	1 426	1 480	950	

*Based on 16-in. core diameter, 20-in. outside shell diameter (2-in. shell required for fire-proofing).

†That is the total permissible column load (by the 1921 Specifications) on the 20-in. diameter column divided by the cross-sectional area of the 16-in. diameter column.

Correcting for the lower strength of commercially fabricated concrete by the factor, 0.6, Equation (59) becomes:

$$\frac{P_m}{A} = 1.20\, N_r\, L_e \left[0.6\,(1 - r_r)(1 + 0.573\, r_s)\, f'_c + 36\,500\, r_r\right] \times \left[1.00 - 0.0183\,\frac{L}{D}\right] \quad (60)$$

$$1.20\, N_r\, L_e = 0.276$$

Table 13 gives permissible stresses by the 1917 and 1921 Specifications and safe stresses as computed by Equations (59) and (60) for comparison.

Criticism of the 1917 Specifications.—

1.—The working stresses permitted by the 1917 Specifications are in disagreement with the safe calculated stresses determined by Equation (60) (in certain limiting cases to a maximum of 33% greater), based on concrete of the strength commercially to be expected from that of a mix of a certain nominal strength.

2.—For concrete of actual strength equal to that selected in design, all the permissible stresses are less than the safe calculated stresses, but the ratio beween them varies greatly with the percentage of reinforcement.

3.—There is only one permissible stress for a wide range of reinforcement and variation in column length, whereas the safe calculated strength varies approximately 100% for the maximum permissible variation in this reinforcement.

4.—The percentage of spiral reinforcement should not be held between the narrow limits of 1 and 2. Although the upper limit of 2% is reasonable, as there are no available data on columns with greater percentages of reinforcement, the lower limit should be less than 1%, and, if limited at all, it should be governed by the minimum diameter of wire that can safely withstand without deformation the handling ordinarily given it.

5.—The allowable stresses may be more than 100% higher than the present allowable stresses for percentages of steel reinforcement larger than those permitted by the 1917 Specifications, but within the limits of reinforcement in test columns.

Criticism of the 1921 Tentative Specifications.—

1.—All the allowable stresses for concrete with an ultimate strength of 1 500 lb. per sq. in. are slightly higher (15% and more) than the safe calculated stresses, where the actual strength of the concrete equals that of the concrete assumed in design.

2.—For the strength of concrete to be expected as commercially fabricated from a concrete mix of a nominal strength of 1 500 lb. per sq. in., the permissible stresses are much higher (to a maximum of 75%) than the calculated safe stresses.

3.—Provided that the actual strength of the concrete of the columns is equal to the nominal strength of 3 000 lb. per sq. in., the permissible working stresses

are less than the safe calculated stresses. The ratio between them is also nearly constant for the permissible range of reinforcement.

4.—For concrete of the strength to be commercially expected from a nominal 3 000 lb. per sq. in., all the permissible stresses are higher, to a maximum of 50%, than the safe calculated stresses.

5.—It is possible to select a range of percentage reinforcement so that the following absurdity occurs: By removing the spiral from a column, the allowable working load is higher by 25% in the example chosen, than that on the spirally reinforced column. The example is, as follows:

Column dimensions: Core, 16 in. in diameter; area, 201 sq. in.; outside diameter of column, 20 in. (a 2-in. shell is required by the specifications for all reinforcement).

Reinforcement: Spiral, 0.25%; rods, 1.00%.

Concrete strength: 3 000 lb. per sq. in.

Ratio: Length to diameter = 10.

The permissible load on this column, if it is considered as spirally reinforced, is 160 600 lb. If the spiral is not considered, the load is 201 600 lb. The removal of the spiral reinforcement will not increase the strength of the column or its safe working load.

6.—The upper limit of the permissible percentage of spiral reinforcement should be 2 instead of 1.25.

7.—No fixed ratio between the percentages of spiral and of rod reinforcement is necessary.

Aggregate Specifications.—It is appropriate to call attention to a suggested improvement in the specifications for aggregate. All criteria may show an aggregate to be defective, yet if the strengths of the concrete test cylinder are acceptable, the aggregate will be accepted. Mr. C. M. Chapman* gives suggestions for the strength of test specimens as the basis of acceptance in place of the tests of the various components. He summarizes the specifications, as follows: The materials used shall be of such quality and such proportions as to produce a concrete which shall show a compressive strength of 2 500 (or 2 000 or 1 500) lb. per sq. in. at the age of 28 days when tested in accordance with the standard methods of testing.

Argument for Present Stresses.—

1.—The actual stress produced in the concrete in a column by the safe working load, as derived in this paper, is, in the extreme case, a maximum of 31.5% of the ultimate concrete strength.

2.—By a process of logical reasoning, resulting in the present stresses for several selections of column variables, these stresses agree with those recommended by the 1917 or 1921 Specifications. If, for several cases, stresses can be calculated entirely from a formula agreeing with the magnitude of the safe stresses determined after twenty years of effort, it is proof of merit in the present method. Consistency, therefore, should make the permissible

* "A New Form of Specification for Concrete Aggregate," *Proceedings*, Am. Soc. for Testing Materials, Vol. XVI, Part II, p. 180.

stresses coincide, or at least bear a fixed ratio throughout, with the values obtained by the methods of this paper.

C.—Structural Steel Columns Embedded in Concrete

The strength of this type of column is a simple summation of the strengths of the two components, the steel and the concrete. For the working stress in the steel, in the present instance, the stresses now used are satisfactory, as it is the writer's purpose to discuss only concrete structures. The stress to be allowed in the concrete will be given by the equation:

$$f_0 = L_e N_r f'_c \qquad (61)$$

the variables having the usual significance. The variables have the values to give approximately,

$$f_0 = 0.3 f'_c$$

Thus, 30% the strength of the concrete, as determined in test cylinders, is the safe working stress for the concrete in this type of structure.

D.—The von Emperger Type of Reinforced Column

The von Emperger type of column presents a case in which the applica- of the methods of determining the safe working stress, as followed in the previous types of columns, would lead to erroneous results. The apparent reliability of the von Emperger column would indicate a safe stress of a single short application of 83.4% of its average ultimate strength, which would mean approximately 31 500 lb. per sq. in. as the working stress in the cast-iron reinforcement. These values for the safe stress in the column and in the cast-iron reinforcement are, however, greatly in error, as will be shown.

The ultimate and maximum strength of the von Emperger column is the sum of two independent quantities: The strength of the spirally reinforced concrete part at the normal strength for concrete thus reinforced, and the strength of the cast iron at the ultimate strain of the reinforced concrete. It is possible that the surrounding concrete would cause the cast iron to assume a larger stress before failure, but until the matter has been definitely decided, the only possible action must be negative.

That the safe working stress cannot be as large as the apparent high reliability indicates, can be proved. The formula:

$$R = \sqrt{r_1^2 + r_2^2}$$

in which, r_1 and r_2 are probable errors of the same quantity due to different operating causes, and R is the actual and final error of the quantity, is a standard and accepted formula in the theory of errors. This formula can be applied in the present case, in which, R is the inverse measure of the strength reliability of the structural compound element, and r_1 and r_2 are similar quantities for the two different components of the compound element.

Let r_1 be the probable variation due to the cast iron and r_2 the probable variation due to the concrete. In the series of test columns, the variation in strength of the cast-iron reinforcement at ultimate column load was not a function of, or variation in, the strength of the cast iron; it depended on the ultimate strain of the reinforced concrete. Assuming that the ultimate strain

of the concrete is constant, r would also be constant, and this indicates that the strength of the cast iron is constant, which is absurd. The tests of the compound element give no measure of the variation in strength of the cast-iron reinforcement, and, therefore, no indication of the safe stress for it.

The safe working stress in the cast-iron reinforcement should not be greater than that used if this cast iron was a separate column without any surrounding concrete. The safe stress for cast iron, as calculated in this paper, is extremely low, about 3 000 lb. per sq. in. As previously stated, this low strength is derived from comparatively poor specimens, and it is probable that tests on vertically cast specimens, such as are now on the market, would indicate much higher safe stresses, even possibly exceeding the safe stress for steel.

Possibly, the embedding of the cast iron in concrete strengthens the cast iron by some stiffening action, preventing any individual specimen of low strength from breaking at a strength as low as it would, if it had not been embedded. To give credit for such increase in strength in the cast iron and permit higher working stresses, based on the ultimate strength of the column, a large number of columns would have to be tested, in order to determine whether a cast-iron core of low strength would reduce the strength of the column, and, if so, to allow for this reduction in the reliability number, N_r.

EXPERIMENTS WITH MODELS OF THE GILBOA DAM AND SPILLWAY

Discussion*

By Messrs. B. F. Groat, Robert Fletcher, I. Gutmann, Robert F. Ewald, and Thaddeus Merriman.

B. F. Groat,† M. Am. Soc. C. E. (by letter).‡—The writer has taken great interest in reading the instructive paper by the authors, and has noted the fact that the overfall did not correspond exactly with their expectations. The question may well be asked—should it? Even if model theory was developed sufficiently to be closely applicable, no one need expect such close agreement that errors will not become apparent. This is true of all theories as regards exact application. In particular, the writer is of the opinion that an overfall is very sensitive to small errors, such as might occur in the measurement of the head on the crest of a model weir.

One element of correct hydraulic model construction which is difficult to create when the prototype does not exist, is a correct introduction of water to the model with corresponding correct configuration of discharge. If these two conditions are not established, an exact reproduction of performance need not be expected. In the case of the writer's model of a reach of the St. Lawrence River, the prototype actually existed, yet, owing to the roughness of the field methods used, errors of some magnitude were observed, not only in the flow within the model, but also in the attempted reproductions of entrance and exit conditions. Had these conditions been minutely adjusted, there is little doubt that a more precise homology would have been presented.

To be even partly true, the statement that the strength of model structures "varies inversely as the scale ratio", must be qualified by the word, "geometrical". When the extended meaning of the word, "model", is used, referring now to combined geometrical and mechanical similarity, no such restricted statement can be applied. In short, the moment the conditions of mechanical similarity are superposed on the conditions of geometrical similarity, the theorem of the statement becomes a "cull". The word, "model", must have an extended meaning or progress in research is at an end as regards models.

In the application of rigorous theory, the construction and testing of a model becomes a large problem. Is a valuable method of solution, however, to be dropped because it is large and troublesome? The chances will be that it is not as large and troublesome as the problem to be solved. The expenditure of

* Discussion of the paper by R. W. Gausmann and C. M. Madden, Associate Members, Am. Soc. C. E., continued from December, 1922, *Proceedings*.

† Cons. Engr., Philadelphia, Pa.

‡ Received by the Secretary, December 19th, 1922.

time and trouble will be justified by the exactness of the results obtained. To make a model structure, for example, one may increase gravity effects by additional density, by loading, by the use of materials of correspondingly varied elastic properties and strength, by centrifugal forces produced by the rotation of the model as a whole, by measuring internal stresses in a model member instead of testing to destruction, etc. Some genius will arise to synthesize materials and fluids capable of being handled with facility in the construction and testing of models. A special study of this kind should be made.

ROBERT FLETCHER,* M. AM. SOC. C. E. (by letter).†—This paper supplies in great detail the actual results looked for by B. F. Groat, M. Am. Soc. C. E., in his paper on "Ice Diversion, Hydraulic Models and Hydraulic Similarity",‡ which the writer discussed.§

All readers familiar with the principles involved must be impressed by the great value of these remarkable demonstrations of what may be learned by the intelligent use of models. In the discussion referred to, the writer objected to what seemed to be too positive and all-inclusive claims for the validity of model studies, and probably over-emphasized the objections without expressing due recognition of the value of such studies when results are rightly interpreted.

The thanks of the Profession are due to Mr. Groat for his very complete exposition of the theory of hydraulic models. It removes some points of haziness, which the writer confesses existed in his own mind. The interesting experiments with models of the Gilboa Dam show how much may be learned practically by the cut-and-try method.

The writer thinks, however, that the main point and intent of his criticism is justified, in all ordinary cases where experimenters have only limited means at their command and are likely to place too much confidence in results which are not conclusive. The final conclusions of the authors from the application of the formulas to the latest experiments are:

First.—A similarity to a certain degree, but not identity, in flow over the actual dam and the model.

Second.—It may be possible to build a model not necessarily a model in the true sense of the word, which "must reproduce even in detail the performances of their prototypes;" but, for the Gilboa Dam, the scale would have to be as 1:2.38, and the liquid must be mercury at 60° Fahr.

Third.—With due recognition of the points of dissimilarity, the model, on a sufficient scale, is the most efficient means of preliminary study of such a case as that of the Gilboa Dam.

No one would presume to dissent from the last conclusion in view of the practical demonstrations made by Mr. Groat and the authors. The second conclusion, however, is an admission of practical impossibility.

If the reason for the difficulties in adjusting the theoretical demonstration to practical conditions is considered, it is noted that it involves inclusion of mass ratio, force ratio, centrifugal reactions (in hydraulics), proportionality of

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† Received by the Secretary, December 20th, 1922.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1168.

§ *Loc. cit.*, pp. 1156–1162.

heads and depths, skin frictions, wind friction, movement and transportation of detritus, in river hydraulics, viscosity, duration of time observations, and factors of safety. Although all these influences are given mathematical expression in the formulas, there are certain coefficients of uncertain value involved, which forbid confidence when an attempt is made to predict what may happen when an extraordinary flood is pouring over a dam, or provision is to be made for a jam of ice or logs, or excessive wave action is to be resisted by a sea-wall, or the impact due to reaction from back-water is to be resisted at the toe of a dam, or in other cases where the forces of Nature are likely to get beyond control. As Mark Twain said, "we are harassed by doubts".

The elaborate German experiments, quoted from the work on river hydraulics* by J. L. Van Ornum, M. Am. Soc. C. E., in the writer's discussion previously mentioned, resulted in great uncertainties and lack of very definite conclusions, although useful in some respects. The artifice of using bags of shot and pieces of slate for groynes and bank protection in the models surely did not realize conformity in mass to materials available for actual constructions on the river.

To be consistent, one must believe that it would be possible to make a model of Niagara Falls, which would reproduce on a reduced scale proportional effects to the erosion and destruction wrought by the mighty cataract, or that officers of the U. S. Ordnance Department could make a model of a 16-in. gun and invent an explosive of detonating effect proportional in the scale ratio to the effect of the actual high explosive used in warfare, and thus predict the safety of the large gun.

In discussing the application of the formulas expressing "the inexorable laws of mechanics" to the model of the Meigs elevated railway, Mr. Groat shows, perhaps conclusively, that, after a certain number of "ifs" had been duly satisfied, "the factors of safety in model and prototype would have been equal", and then adds: "It is not doubted that it would be something of an undertaking to construct and test such a model, but the fact which we have proved is that it can be done." Again, the verdict is, not really practicable because of inherent limitations.

Acknowledging the great value of all the experiments described, having specific ends in view, the writer would disclaim any intention, both now and heretofore, to under-estimate the value of the judicious use of models. The caution is against the misuse of them, especially by inexperienced experimenters.

I. Gutmann,† Assoc. M. Am. Soc. C. E. (by letter).‡—In 1919, the writer was instructed by G. A. M. Elliott, M. Am. Soc. C. E., Chief Engineer of the Spring Valley Water Company, of San Francisco, Calif., to prepare a tentative design for a spillway for the Calaveras Dam. The spillway was to take care of a drop of about 200 ft.

An extensive preliminary study, covering more than seventy reservoir spillways in all parts of the world, was made in connection with this design. Spe-

* "The Regulation of Rivers," pp. 163–172.

† Brooklyn, N. Y.

‡ Received by the Secretary, December 30th, 1922.

cial attention was given to stepped spillways or cascades, that is, the type used on the Gilboa Dam.

Stepped spillways and wasteways have been used in European* and American† practice for many years, yet the principles for their design have not been formulated, and the writer had to state that there was no reliable formula for the flow of water over cascades. The absence of water-cushions and the consequent progressive increase in velocity of approach preclude a satisfactory analysis. At low heads, each step may be treated as a drop, with no raised crest, and the formula, $Q = 4.75\ L\ H^{\frac{3}{2}}$, may be used. German engineers prefer the Kutter formula. At high heads, with the water being projected farther horizontally and not all the nappe striking the next step, the problem will become more complex and the formulas preposterous.

The writer has been especially interested in this valuable paper which presents the results of the authors' experiments to determine the conditions of flow over the stepped overfall dam at Gilboa, N. Y., by means of a set of models.

Experiments with models could not be expected to furnish very accurate results in this particular case which is essentially dynamic and involves so complex a phenomenon as the successive impingements of a sheet of water of varying head on masonry steps of varying width and height. Considering the 1:8 model (Fig. 7‡), as a prototype, and the 1:20 and 1:50 models as its 1:2½ and 1:6¼ reductions, one will conclude that it will require unusual intuition to exterpolate the curve of flow over the prototype from observations on the models, especially in the case of ordinary heads, namely, less than 4 ft. or more.

More consistent and, therefore, more serviceable results were obtained with the models of the "approved Section 100" (Fig. 9)§. It may be said, perhaps, that Section 100 was approved because its models gave more consistent results, these results being probably due to the greater uniformity of its profile. The rises of the steps in Section 100 are consistently greater than their treads, whereas, in the experimental sections, the rises are smaller than their treads, in the uppermost three steps and greater in the steps below the third, therefore, in Step 2, the ratio of rise to tread is 1:1½, and in Steps 5 and 6, the ratio is 2:1.

Most of the "discrepancy" or "dissimilarity" with the experimental models probably originated on Step 2, the tread of which was long enough to deflect and to project the falling nappe in a horizontal direction, whereas the profile of the model, near that point, takes a vertical deflection downward with reference to its original trend. The flow curve diverges from the profile, and the dissimilarity between the models, which becomes prominent below this point,

* Crosswood and Roseberry Reservoirs in Scotland: also, at Sengbach, Urft, Bystricka, Krauserbauden, Möhne, etc., in Germany and former Austria-Hungary; and at Rochebut and others in France.

† Titicus, New Croton, and others of New York Water Supply; also, the Lahontan Reservoir of the Truckee-Carson Irrigation Project.

‡ *Proceedings*, Am. Soc. C. E., September, 1922, p. 1515.

§ *Loc. cit.*, p. 1517.

is probably due chiefly to the mass effect of their respective volumes of water which had impinged on and were deflected by the tread of Step 2.

In the "approved Section 100", the horizontal deflection of the nappe, taking place on Step 2 of the experimental sections, was suppressed by shortening the tread of Step 2 and by a general steepening of the upper part of the profile of the dam. This made the profile more uniform in respect to rise and tread proportions, the structure becoming geometrically simpler and the results more consistent.

Better results were obtained by the authors in their tests of models of broad-crested weirs, the obvious reason being the greater simplicity of the structures and the absence of the complex impingement factor. In connection with these tests, the authors made the interesting observation that kinetic dissimilarity may be eliminated "by changing the geometrical similarity to a slight degree, that is, by using a corrected head, flows may be obtained which are identically similar for all models",* irrespective of their scale. The corrections computed by them for the three models used seem to be consistent when properly plotted against the scales of the three models. Curves based on such observations might supply a very useful link for the correlation of observations on models with what may be expected to occur on full-scale structures.

In general, the study tends to show that the method of models, if rationally and cautiously used, can be reasonably depended on even in quantitative studies of proposed broad-crested weirs and other hydraulic structures not unduly complicated.

The authors deserve great credit for their thorough investigation and their valuable contribution to hydraulic literature.

ROBERT F. EWALD,† ASSOC. M. AM. SOC. C. E. (by letter).‡—There is much to commend and little to criticize with respect to the methods used in and the conclusions obtained from the admirable series of experiments made on models of the stepped profile type of dam. The general type of dam to be used apparently had been decided on before the experiments were undertaken, and as no mention is made in the paper of tests on smooth-faced ogee spillway dams, it appears that such a dam was not seriously considered, owing no doubt, to the apparent difficulty of properly absorbing the high velocities of the water at the toes of dams of that type. In the light of knowledge gained during the past ten years, the apparent difficulty has disappeared, and as there are certain inherent serious disadvantages in the stepped type of overfall dam, there are strong reasons for believing that a series of experiments as ably conducted should have been carried out to determine physically the feasibility of a smooth-faced ogee dam before the stepped profile was adopted.

The writer has never been enthusiastic over the stepped type of spillway dam, notwithstanding its occasional use in the northeastern part of the

* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1525.

† Asst. Engr., Aluminum Co. of America, Pittsburgh, Pa.

‡ Received by the Secretary, January 5th, 1923.

United States. It is not easy for him to get away from the idea that the absorption of hundreds of thousands and possibly millions of horse-power of energy by "pounding it out" on the dam itself cannot be done without immediate serious destructive action on the dam and within areas where such action is least desirable and most dangerous.

Cyclopean and coursed masonry and even concrete deteriorate considerably in time insofar as freedom from cracks is concerned. Wide variations in temperatures and to some degree in stresses, due to loads, ultimately develop cracks, possibly very minute but, nevertheless, present and extending to considerable depths. This tendency to crack is considerably facilitated by the greater exposure and angles introduced by the stepped construction and by the entrance of water under direct impact, which takes place on every step. It can be only a relatively short time before the corners of the steps are broken away, due to weathering and impact. After this has happened, it is highly probable that the exceptional flood will come and the steps, now no longer of sufficient width to function properly, will then become a source of positive danger. In the same manner as discussed in the experiments, where the steps were not made sufficiently wide, some of the water with its velocity unbroken will reach the lower parts of the face of the dam with sufficient energy to develop erosive action which the lower steps cannot withstand for any appreciable length of time. The result will be rapid destruction of the steps and serious attrition of the main body of the dam near its base where the stresses are greatest and where adequate repair will be most difficult if not impossible to make.

The great rapidity with which raveling action proceeds where water is flowing at high velocities over or directed against checked and cracked surfaces, especially where these surfaces are irregular, is familiar to many engineers. The writer has in mind as a particular case the tremendously destructive action of water flowing at a velocity of about 45 ft. per sec. in a spillway channel newly excavated in a very hard rhyolite formation. The surface as left by the contractor was rough and broken, but as blasting had been very light in order to prevent as far as practicable the breaking of the rock left in place, it is difficult to believe that fracture at depths greater than 10 ft. could have been more than what one would term "hair cracks", yet in the course of a few days during the first big flood discharging through the channel, holes 15 ft. or more in depth were scooped out of the ledges in place and blocks of rock many tons in weight carried down the channel. The average slope of the channel was about 30 ft. vertical in 100 ft. horizontal, and the maximum depth of water in the channel at the beginning of the flood was about 7 ft.

Water may be allowed to flow over smooth concrete at very high velocities for a long time without appreciable wear, but where slight irregularities develop, erosive action appears and increases with great rapidity. The initial irregularities need be very slight to start the trouble. The writer has recently inspected some photographs of the down-stream toe of an ogee spillway dam, over which water has been flowing with velocities of more than 100 ft. per sec.

and in these photographs there is evidence that wear is beginning at the horizontal joints marking different pourings of concrete, and it is safe to say that the erosion at these points will develop rapidly. These bonding joints were made with the greatest care and the lines of division must have been microscopic, nevertheless, they appear to be sufficient to give the water its initial "grip". On the adjacent areas, originally smooth and unbroken, the wear is very slight.

The statement no doubt will be made that the steps can be repaired and renewed from time to time, but this involves a heavy maintenance charge and if not done may ultimately result in trouble, as it is not certain that those in charge of the dam in the future will appreciate the necessity of keeping these steps in perfect repair. It is doubtful whether the repaired sections would have the length of life of the originals, as it is extremely difficult to bond new and old concrete or masonry in such a manner that water flowing at high velocity will not soon begin to erode the joints. Furthermore, the filling of holes with concrete so that microscopic joints can be avoided is practically an impossibility, owing to the tendency of the filling concrete to shrink away from the hole in which it is cast.

As previously stated, during the last few years much knowledge has been acquired regarding the conditions under which high velocities can be absorbed successfully or controlled by water-cushions and the burden of acting as a buffer shifted from the dam to areas more remote from the vulnerable parts of the structure and to construction more readily and satisfactorily repaired when the inevitable wear makes such action necessary.

Early in 1914, the writer performed a fairly elaborate series of experiments on models of an overflow dam to be constructed on the Little Tennessee River, about forty-five miles south of Knoxville, Tenn. The prototype was to be 195 ft. high and the models were on a scale of 1: 48. The conclusions drawn from the experiments were summarized in a report made to the Knoxville Power Company, in June, 1914, the more essential paragraphs of the conclusions being as follows:

"Practically all of the kinetic energy of water flowing at high velocities in open channels may be internally absorbed by directing the water into pools of water exceeding a certain minimum depth and a certain minimum length.

"The theoretical minimum depth of the pool is given by the equation: $d_2 = \sqrt{\frac{d_1 \, V_1^2}{2\,g}}$, in which d_1 is the depth of the water before entering the pool and V_1 is the velocity of the water at the same point. This equation gives the required depth of the pool, d_2, above the same horizontal datum above which d_1 is measured.

"The required length of the pool is at least approximately equal to the net velocity head absorbed by expansion divided by 1.35. This gives the distance in which the bottom velocity is completely absorbed. About 20 per cent. should be added to allow for complete absorption of velocities above the bottom.

"In high dams, subject to high velocities at the toe, complete absorption of the high velocities within the shortest distance possible is desirable to prevent channel erosion and turbulent conditions in tail-races, as much as possible.

"An efficient method of absorption is by directing the water into pools having the minimum depth given in preceding paragraphs.

"The height of the lip of the bucket above the bottom of the pool has little effect on the general efficiency of the absorption if the lip is kept within the lower half of the pool's depth. The bucket with the lowest possible lip will allow the least wear on the concrete, but as this is negligible on a well designed and constructed dam, the advantages of the highest possible bucket in decreasing the height through which the water falls and also allowing the use of longer buckets along the axis of the dam with less excavation and less concrete may be considered paramount. The highest possible position of the lip with present knowledge is perhaps about half the depth of the pool.

"The required depth of pool varies in the same sense as the depth of water on the crest of the dam, hence the longer the crest the shallower the required depth of the pool. The bucket must of course also be made longer.

"For the same head on the crest, arched dams, by increasing the value of d_1 for the same discharge, require greater depths of cushion pools than straight dams.

"Where pools of sufficient depth are not provided, it is probable that the entire kinetic energy of the water will be expended upon the channel and erosion will continue at a rapid rate until a pool of sufficient depth is excavated by the water."

During 1916, and, subsequently, the Miami Conservancy District made elaborate experiments that clearly demonstrated the efficiency of the hydraulic jump as a means of absorbing high velocities. The data and conclusions were published as Part III of Technical Reports, under the title, "Theory of the Hydraulic Jump and Backwater Curves and the Hydraulic Jump as a Means of Dissipating Energy". Mr. R. D. Johnson, in his discussion* of the paper, by Karl R. Kennison, M. Am. Soc. C. E., entitled "The Hydraulic Jump, in Open-Channel Flow at High Velocity", and in a paper† on "The Correlation of Momentum and Energy Changes in Steady Flow with Varying Velocity and the Application of the Former to Problems of Unsteady Flow on Surges in Open Channels", presents in an excellent manner the mathematical and physical conceptions involved in the hydraulic jump and the related hydraulic feature, the standing wave, and casts aside the shroud of mystery with which the every-day engineering world seems to have covered this really common and natural occurrence. It is probable that the theory of the hydraulic jump and its practical application are now fairly well understood and that few dams of any magnitude are constructed without consideration of its development and whether or not artificial means of insuring the jump will be required now or in the future.

The writer considers the ideal spillway dam to consist of a perfectly smooth-faced ogee dam, straight in plan, with the crest unbroken and designed to conform to the lower nappe of the falling water for the highest possible flood; the down-stream face of the dam to be joined to the floor of the channel by an easy curve; the channel down stream to be the same width as the crest of the dam for a distance sufficient to permit the full development of the normal jump (farther down stream the channel may be varied to suit conditions stated subsequently); the floor of the channel near the dam to be nearly level throughout, and as its position vertically is defined by the low point in the

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 378.

† *Engineers and Engineering*, July, 1922.

natural channel, this requirement also agrees with the one that the water should be taken through the entire fall in a single drop along the entire crest. Finally, the hydraulic functions of the channel down stream from the dam should be such as to hold the jump at the toe of the dam. Natural conditions may suffice, if not, secondary dams may be used, or the channel may be deepened artificially or be permitted to deepen naturally, as it will if sufficient depth is not provided originally.

To develop the ideal spillway economically, the dam would have to be built in a U-shaped valley. This is rarely possible, especially where very high dams are involved, the valley usually being V-shaped with respect to the dam. To meet as nearly as possible the ideal requirements, economical construction then suggests widening a certain part of the bottom of the V to permit the dropping of as much as possible of the water for the full height to provide for frequent floods and placing gates on the crest of the dam to hold the flow as much as possible within the length of crest discharging its waters directly into the cushion pool. The remainder of the crest is to be opened to flow only as required. The stepped profile may be used in these sections, or if the foundation rock is of such quality as to resist erosion well, the water from an ogee spillway may be turned over it. In either case, however, repair work from time to time will be necessary.

If gates are not permissible on the crest, a rather difficult problem is presented. If water is spilled for a considerable part of each year, the best solution seems to be in smoothing the rock surface and armoring it with concrete, the lower end of the slope to be treated the same as an extension of the dam and the water guided horizontally into cushion pools by easy vertical curves. In any case, the direct impact of water at velocities in excess of, say, 10 ft. per sec., is to be avoided and all the water guided by surfaces made as smooth as possible into ample cushion pools.

In the cushion pool itself, very little armoring is required, as the energy is absorbed entirely by the water. Mr. Johnson has raised the interesting point that under ideal conditions in the hydraulic jump there probably is a certain quantity of water "humped up" at the bottom of the jump to form a natural weir. The inference is then that this water is practically "dead". If this is true, then high velocities are absent immediately above the channel floor and, therefore, erosion on surfaces on which the jump actually takes place is practically absent. This means that if the initial point of the jump is held on the concrete at the toe of the dam, erosion down stream from this point need not be feared, as long as the surface of the bucket and the floor of the channel are at the same elevation and the jump is actually formed. The writer is inclined to agree with this proposition. His own experiments were not sufficiently detailed to give any information on this subject. A few experiments recorded by the Miami Conservancy District, shown on page 79 of the published report, seems to confirm Mr. Johnson's theory. Several of the writer's experiments were made on models on which the horizontal edge of the bucket was about 2 in. above the floor of the channel. A favorite test was to fill the channel with sand level with the edge of the bucket and then start the water over the model after making sure that the jump would be developed. Under

uniform flow conditions, the sand for several inches down stream from the bucket would not be moved, although velocities of 14 ft. per sec. prevailed within 1 in. of the edge of the bucket. Eddies, however, could be produced by slight manipulations that would sweep out all the sand so the experiments were not conclusive.

With regard to the Gilboa Dam, it would be admittedly somewhat difficult to utilize readily the hydraulic jump throughout the entire length of structure, but it appears from such data as are available that at least two-thirds of the dam could have been built as a smooth-faced ogee spillway without difficulty and the water from this part of the crest directed into a cushion pool. It is reasonably certain that a far more efficient method of absorbing the high overfall velocities would have been utilized and the chances of future trouble greatly minimized. With a carefully designed crest, using the same free-board specified for the dam actually being built, the crest length for a maximum discharge of 110 000 sec-ft. could have been reduced to 900 ft., assuming a weir coefficient of 3.90 which is reasonable. At the maximum section, the discharge would be 121.5 sec-ft. per foot of crest, the velocity of the water at the toe of the dam about 98 ft. per sec. and the depth, at the section at which this velocity exists, 1.24 ft. According to Unwin's formula for the jump, the required depth of water to develop the jump for these conditions would be 24.8 ft. per sec., and the writer is inclined to believe that the natural flow conditions down stream from the dam would easily maintain this depth. If not, a low dam not less than 140 ft. from the main dam would be required.

Owing to the much higher elevation of the natural ground at the base of the dam toward its ends, not more than 600 ft. of the shortened crest could be protected economically by the main cushion pool. The discharge for the remaining 300 ft. of crest would have to pass into cushion pools set stepfashion at higher levels and of much more expensive construction, or down smoothed and armored slopes into the main pool, or the velocity of the water destroyed by use of a stepped profile for the dam, the actual method to be used to be developed by comparative studies.

The writer hopes soon to publish data on the conditions at the toes of two overfall dams 200 ft. high, completed in 1917 and 1919, respectively, which have been subjected to several floods since their completion. In certain parts and at certain times, natural water depths have not permitted the jump to develop, and high velocities have occurred resulting in considerable erosion. In time, natural cushion pools will be formed below these dams, and reasonable protection will then be obtained against continued erosive action without secondary dams. In any case, he hopes more information will be derived on the interesting problem of endowing high overfall dams with the longest possible life.

Thaddeus Merriman,* M. Am. Soc. C. E. (by letter).†—The writer has long been convinced that the use of accurate scale models in the solution of hydraulic problems involving dynamic effects will give better results than can

* Chf. Engr., Board of Water Supply, City of New York, New York City.

† Received by the Secretary, January 10th, 1923.

possibly be secured by abstract theoretical considerations. During the greater part of the work so well described by the authors, he was in close touch with the experiments conducted by them and will refer briefly to the cause which he believes to be responsible for the dissimilarity referred to in the paper as having been observed in the behavior of models of different scales.

Water on approaching a weir crest has one free surface; but immediately after passing the crest, it has two such surfaces. For the same length of weir, irrespective of its crest width and of the depth of flow, the increase of free surface will be the same. As surface tension as between water and air is a constant for each unit of area, it follows that, when the surface is suddenly doubled, energy must be expended. The amount of energy expended in the formation of the under side of the nappe is proportional to its surface area and as this area, in turn, is proportional only to the weir length, it appears that the energy required for the formation of the under side of the nappe is not proportional to the scale of the model. It follows as a necessary consequence that models of different scales will not exhibit the same characteristics for all depths of flow over them. The thinner the sheet the greater will be the divergence from true similarity of behavior. This difference is clearly shown in Fig. 13,* where the divergence for the proportionate depth of 3 ft. is materially greater than that for the proportionate depth of 6 ft.

The differences observed are due to the fact that the energy required to form the under side of the sheet is a constant for each unit of crest length. The thinner the sheet the greater will be the ratio between the energy required for its formation and the total energy in the vertical plane of the weir crest; that is, the energy remaining after the nappe has been formed will be least, per unit volume of the sheet, for the thinnest sheet, and, in consequence, the thinner the sheet, the less far will it be projected. Approaching the limit of zero thickness the nappe, instead of being projected beyond the crest, would either fall straight downward or be pulled inside the vertical. This is a phenomenon which the writer has often observed, but which is much complicated by the surface tension as between the material of the weir crest and the water which usually has a different value from that between the water and the air.

This factor of surface tension is noticeable only at shallow depths of flow, and it is only at such depths that marked dissimilarity is observed. The differences which were noted at greater depths are those naturally incident to errors of observation, which include not only the personal equation of the observer, but also differences due to changes in the direction and intensity of the wind as well as to errors in the setting of the measuring apparatus which for any one series are generally constant.

It is of interest to note that the value of the surface tension as quoted by Besant† is 7.83 mg. per mm. of length. This tension, if multiplied by the rate at which the nappe is being formed, will represent the energy required for the formation of the sheet per millimeter of its length. This energy is a small proportion of the total energy of the system and for any appreciable thickness of the nappe is entirely negligible. At low heads, however, its effect becomes noticeable.

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* *Proceedings*, Am. Soc. C. E., September, 1922, p. 1521.

† "Elementary Hydrostatics", Cambridge, 1892.

THE COMPARISON OF CONCRETE GROINED ARCHES AS AN AID IN THEIR DESIGN

Discussion*

By Thomas H. Wiggin, M. Am. Soc. C. E.

Thomas H. Wiggin,† M. Am. Soc. C. E. (by letter).‡—The groined arch of concrete in filter and reservoir roofs, was fairly well standardized in the period from about 1898 to 1910, when the large slow sand filter plants at Albany, N. Y., Philadelphia, Pa., Washington, D. C., Pittsburgh, Pa., and Springfield, Mass., as well as a number of smaller plants, were being planned and built. Probably few engineers omitted the rather farcical process of drawing lines of resistance, but a crown thickness of 6 in. appears to have resulted in all cases except where unusual conditions existed, such as an additional structure to be supported by the groined roof. Spans, rises, and depth of depressions over piers were and are subject to considerable variation, the tendency being toward longer spans, and the author's tables and diagrams will be useful in showing the dimensions in many existing plants and the relative economy of various combinations of span, rise and depth of depression over pier, as shown by figures giving average thickness of roof.

A table giving the dimensions of twenty-seven groined roofs, prepared by Leonard Metcalf, M. Am. Soc. C. E., was published in 1903.§ This table was extended by Morris Knowles, M. Am. Soc. C. E., and, again, by the writer, and is included in the writer's paper entitled "The Concrete Groined Arch".‖ On page 1666,¶ the author refers to a condensed report of this paper. This table, as extended, gives data on forty-four groined arch roofs, about twelve of which are enumerated in the author's list of thirty in his Table 1. Data on about sixty-two roofs, therefore, are available in the two tables. The writer's table also gives data on floors for fifteen of the forty-four cases.

The author's Paragraph (*b*), on page 1669,** explains in general terms the effect of spacing and height of piers on the design of roof. It would be useful if he could include a table or diagram showing quantitatively the effect of span. The comparative tables and diagrams relate to a single span, namely, 17.96 ft., and the statement, on page 1671,** that "it will also be found that the average thickness decreases directly with the span", is not apparent from

* This discussion (of the paper by Philip O. Macqueen, Assoc. M. Am. Soc. C. E., published in October, 1922, *Proceedings*, but not presented at any meeting of the Society), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr., New York City.

‡ Received by the Secretary, December 21st, 1922.

§ *Journal*, New England Water Works Assoc., December, 1903, p. 400.

‖ *Proceedings*, National Cement Users Assoc., 1910, p. 246.

¶ *Proceedings*, Am. Soc. C. E., October, 1922.

the examples from practice given in Table 1. Thus, compare Lines 21 and 30 of Table 1, as follows:

Line.	Clear span (2a), in feet.	Rise of intrados (b), in feet.	Crown thickness (t), in feet.	Rise of extrados (h), in feet.	Span, center to center of columns (2c), in feet.	Ratios.		Average thickness of roof, in feet.
						$\left(\frac{b}{2a}\right)$	$\left(\frac{h}{b+t}\right)$	
21	14.08	3.50	0.50	2.00	15.75	0.249	0.500	0.618
30	17.96	4.50	0.50	2.50	20.29	0.253	0.500	0.675

In Line 21, the pier capital is 1.67 by 1.67 ft., the area of which is $\frac{1}{88}$th of the roof area; in Line 30, the pier capital is 2.33 by 2.33 ft., the area of which is $\frac{1}{76}$th of the roof area. The relatively large pier capital in the case referred to in Line 30 evidently had more effect on the quantities per square foot than the span.

To make the comparison complete, quantities in piers should be included, as the author states; when the height of the roof at the crown is one of the fixed quantities, as is sometimes the case, it is satisfactory to do so, as quantities in side-walls, in that case, are not affected by rise of the arch. In some cases, head-room near the piers or the assumption that the water level shall not be above the springing line, complicates the comparisons.

Quantities in floors should also be included in economic comparisons unless the reservoir is founded on rock so that the floor does not have to support the pier load. Analyses of stresses in groined floors are even less satisfactory than those in roofs.

In the interesting pursuit of theoretical economy, it is easy to forget some practical limitations. Thus, the depression over the pier, h in the author's tables and diagrams, increasing the depth of which is so important in securing economy of material, is a trouble in construction. The deeper this depression relative to the span, the more difficult it is to keep the concrete from sloughing into the depression, also, the greater the chance of unintentionally robbing the arch of thickness at the haunch and the dryer the concrete must be. Furthermore, ramming on the slope makes the concrete slough into the depression. Forms for the depression have been tried, but only experimentally as far as the writer knows. Concrete workers on groined arches become surprisingly skillful in forming these depressions, yet it would doubtless be true economy to limit the steepness of the surface slope. It would be interesting to see a line of practical limitation (similar to the deflection limit in a table of floor-beams) added to the tables and diagrams on the basis of surface slope. A contractor at his own expense would generally prefer to fill exaggerated hollows somewhat. Probably, it would be better to assume a tangent of a certain slope at the lower end of the parabola of the extrados instead of extending the parabola to the bottom of the depression. The computation for quantity would be less easy. Allen Hazen, M. Am. Soc. C. E., described the depression in roofs of the Albany filters as a cone and not a pyramid.*

* *Journal*, New England Water Works Assoc., December, 1903, p. 408.

The writer's paper, to which the author refers, contains this sentence: "The groined arch, although used centuries ago, remains a purely empirical type of structure, designed, or speaking more accurately, drawn, without satisfactory mathematical analysis and used without much knowledge of the factor of safety." The author takes a similar view and concerns himself with compiling aids for empirical design. The writer treated of certain other theoretical and practical problems in groined roof construction, that are related to design and perhaps may be briefly mentioned.

At the edges of a filter or reservoir, the thrust of a groined arch is often opposed by the thrust of a side-wall having a section about like one-half an ordinary horseshoe-shaped aqueduct. The "active" thrust from this side-wall is much less than that of the groined arch which is comparatively flat. Manifestly, the "passive" resistance of the earth against the side-wall is called into play, or the groined roof supports itself partly by cantilever action. Shrinkage and change of temperature of the roof are sufficient to cause the construction joints along the crown to open, sometimes as much as $\frac{1}{8}$ in. One who has designed arch bridges by rigid methods, in which the distortions due to stresses and temperature are consistently accounted for, knows that a short arch, even if it is without groins, could not take up such a shrinkage without cracking. When the ties afforded by the straight elements of the arches at right angles are considered, it is evident that, during the colder weather, it is only after the formation of vertical cracks along planes approximately joining piers that the groined roof can act as a rough voussoir arch. The writer has observed many such cracks. These cracks are not dangerous, unless they happen to occur on a slope such as to release a section of roof and allow it to fall. It is only this latter occurrence, which happens very infrequently, that would deter the writer from using arches 3 or 4 in. thick in some situations. The greater the rise, the less the danger that pieces may be released. Doubtless, the load carried by arch action through the earth covering, or, perhaps, by beam action in frozen earth, relieves the concrete of considerable stress.

It was formerly thought that groined arches could carry the load without arch action, that is, by cantilever action. The idea was suggested by the safety with which wagons and horse-drawn rollers could move near the free edges of partly constructed groined arch roofs. Computation of these loads, however, showed comparatively light concentrations. Furthermore, strips of roof were constructed monolithically and a traveling load would be supported by several groins aiding each other. Foremen's stories of long unsupported edges of groined arch roofs, covered nearly to the edge with full depth of earth fill, could never be substantiated by the writer. In order to obtain an idea of the cantilever strength of concrete groined roofs where they are not supported by arch action, several isolated groin units, symmetrical about a pier, were built and tested in 1905 by the Bureau of Filtration of Pittsburgh, Pa., of which Morris Knowles, M. Am. Soc. C. E., was then Chief Engineer. The work was done under the direction of the writer, then Division Engineer. W. A. Bassett, M. Am. Soc. C. E., was Assistant Engineer in immediate charge of the construction and testing. These tests have never been published and by arrange-

ment with Mr. Knowles are presented below as a part of this discussion. Most of the data are from Mr. Bassett's final report dated February 13th, 1906.

Nine groin units, each 15 ft. square, were tested. Fig. 5 and Table 5 give the dimensions, breaking loads, and deflections of these groin units. Fig. 6 gives sketches of the lines of fracture for Groins Nos. 1, 2, 4, and 6. Other data are as follows:

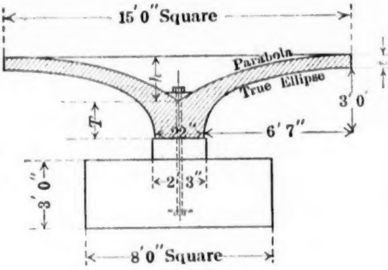

FIG. 5.

TABLE 5.—DIMENSIONS AND TESTS OF 15 BY 15-FT. GROIN UNITS.

Groin No.	Thickness at crown, in inches.	Thickness over pier, t, in inches. ‡	Age, in days.	Deflection at edge, in inches. §	Breaking Load Exclusive of Dead Weight and of Sand Used in Leveling. Total, in pounds.	Pounds per square foot.
1	6	21	41	0.002	19 585	88
2	6	21	39	0.002	53 985	240
3	4½	19½	Broke on removal of form. Concrete 6 days old.			....
4	3	18	Broke on removal of form. Concrete 18 days old.			....
5	6	21	39	0.000	13 000	58
*6	6	21	35	0.005	50 700	225
7	6	21	40	0.004	30 000	133
*8	6	21	40	0.005	39 900	178
†9	6	21	40	0.003	33 300	148

* Reinforced with Thatcher rods.
† Aggregate composed of sand and blast-furnace slag, proportions, 1 : 2¼ : 5¾.
‡ Rise of extrados, h, is 21 in., or 1.75 ft. for all nine groins.
§ Deflections are not regarded as very reliable. They do indicate that deflection was small.

Foundations.—Blocks of 1 : 3 : 7 concrete, 8 ft. square and 3 ft. thick, were built in excavations made in the fill of blast-furnace slag at the "Isabella" furnaces near Pittsburgh. The fill was firm, but subject to slight jar from trains passing on adjacent freight tracks.

Dimensions of Groin Units.—The bearing on piers was 22 in. square, leaving a clear semi-span of 6 ft. 7 in., so that the full span would be 13 ft. 2 in. The rise was 3 ft. in all cases. These dimensions were the same as those used in constructing the filter plant and are given in Line 14 of the author's Table 1. The crown thickness was in general 6 in., but Groins Nos. 3 and 4 had thicknesses of 4½ in. and 3 in., respectively. The depth of depression, or rise of extrados as the author names it, was 1.75 ft., but the bottom of the depression was filled in slightly to a flat surface to make a bearing for the anchor-bolt.

Groins Nos. 6 and 8 were reinforced with a single ¾-in., Thatcher reinforceing bar, which was placed near the outer edge at about mid-height in the 6-in. thickness. It was, therefore, in the form of a square, but the corners were rounded. (See plan of fracture of Groin No. 6 on Fig. 6.)

Cement Used in Groin Units.—Lehigh cement was used and the standard tests of briquettes gave the results shown in Table 6.

Aggregates Used in Concrete for Groin Units.—In general, sand and gravel dredged from the Allegheny River were used. In one case, namely, Groin No. 9, the gravel was replaced with blast-furnace slag, a greenish, vitreous, somewhat porous material, much lighter than gravel. Allegheny River sand is

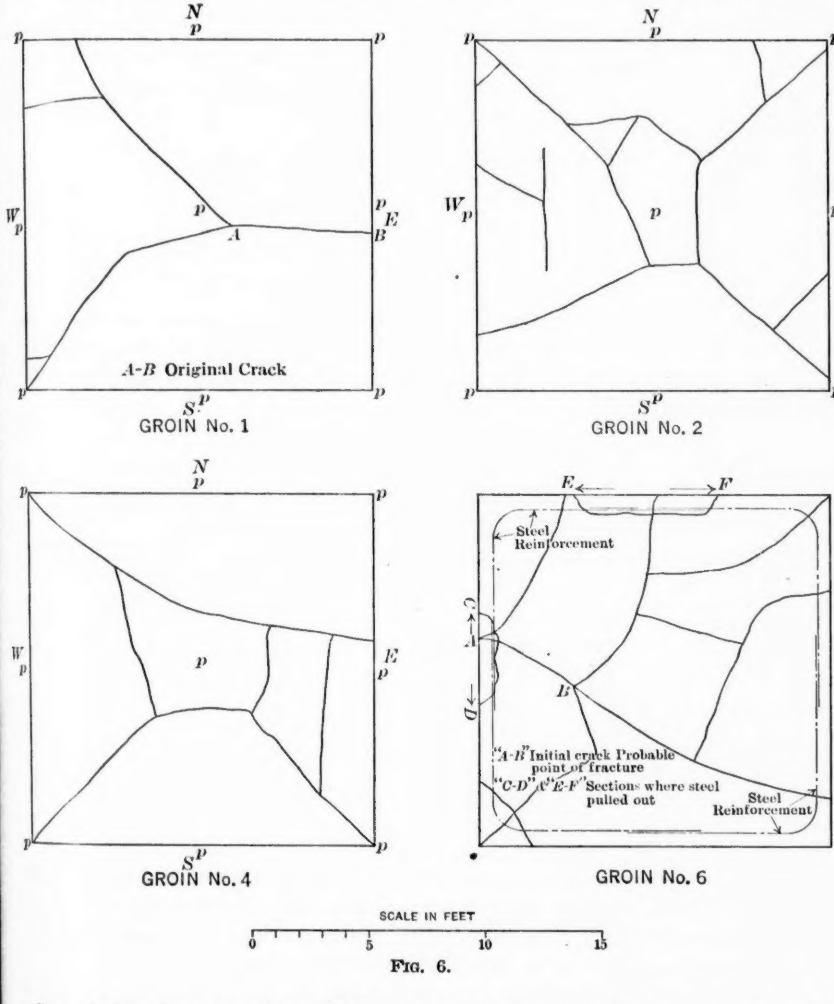

FIG. 6.

well graded and not too fine. The writer, unfortunately, has no records of the mechanical analyses available. The gravel is sound and hard, but rather flat, and is well graded from fine to coarse, the larger stones being about 1½ in. across.

Quality of Concrete in Groin Units.—The concrete was mixed by hand, but thoroughly. Its strength is indicated by the tests given in Table 7, of beams,

6 in. square in section, made with concrete mixed and ready to place in the groins.

TABLE 6.—TESTS OF CEMENT.

	Neat.		1 : 3	
	7 days.	28 days.	7 days.	28 days.
Samples Representing Whole Shipment of 145 Bbls:				
Average	735	869	260	350
Maximum	1 025	1 065	305	395
Minimum	365	675	215	290
Samples from Cement Used in Particular Groins:				
Groin No. 1	....	....	200	280
" " 3	750	900	175	300
" " 5	700	700	225	325
" " 8	620	563	223	305
" " 8	706	625	233	281

Groins Nos. 1 to 8 were made of 1:3:5 sand and gravel concrete, and Groin No. 9 was made of 1 part cement to 2¼ parts sand to 5¾ parts blast-furnace slag. Much water was required for this latter mix, possibly on account of the porosity of the slag.

TABLE 7.—TESTS OF 6 BY 6-IN. BEAMS MADE FROM CONCRETE MIXED FOR GROIN UNITS.

Concrete from Groin No.	Age, in days.	Span, in inches.	Center load, in pounds.	Modulus of rupture, in pounds per square inch.
1	287	30	1 260	264
2	294	30	2 598	557
2	297	60	1 098	502
3	299	30	2 653	581
3	309	30	2 506	502
3	312	60	903	401
5	220	30	1 101	252
5	220	60	382	194
6	196	30	1 247	258
7	195	30	1 376	294
8	163	30	1 474	314
9	180	30	1 003	213

The groin units probably did not set as rapidly as they would have in general in the filters, because there was no drain from the depression, and the early summer rains kept them filled with water much of the time. The concrete, however, appeared to be fairly well set and strong.

Tests of Groin Units.—Slag was piled under the groins leaving a space of only about 3 in., in order that the impact of the fall might not break them into smaller pieces and disguise the lines of fracture produced by the loading. In spite of this precaution, it is known that many of the cracks shown on the diagrams of fracture, in Fig. 6, resulted from the fall. The fall was difficult to watch, but it is known that the initial break in nearly all cases took place near the center of a side. However, Groin No. 2 which was the strongest,

apparently broke first nearer a groin line than the center of a side. Groins Nos. 1 and 5 cracked before any load was applied, owing perhaps to shrinkage stresses or jarring, or to a combination of these causes. Final failure took place at the same cracks under a light loading. Groins Nos. 3 and 4, respectively $4\frac{1}{2}$ and 3 in. thick at the crown, fell down when the forms were removed, which was done at 6 days and 18 days, respectively.

Iron, in pigs weighing about 100 lb. each, was used in the loading. These pigs were piled corncob fashion on isolated wooden bases, 22 by 24 in., symmetrically placed on the level bed of sand with which the depression in the groin had been previously filled. Therefore, no tying strength was afforded by the load itself.

It appears to be a safe conclusion that groins made of good concrete and supported on piers about 15 ft. apart, will about sustain a fill of 2 ft. of earth without developing arch action. As the groins are generally concreted in strips, the strength of part of the roof would be greater, owing to the effect of continuity.

The effect of steel reinforcement was not fairly measured by tests of Groins Nos. 6 and 8, because the failure took place in the region of the lap, and the steel pulled out. In Groin No. 8, the steel was flattened, drilled, and fastened together at the splice with a number of $\frac{3}{8}$-in. bolts. The steel broke at a bolt hole. It would probably have been better to put the laps at the corners, although the best groin—No. 2—broke at and near the corners. Computations indicate that such a small quantity of steel would not be very effective in any case. Cantilever action of groined roofs is unnatural, being inconsistent with arch action. Steel placed to aid cantilever action would be of use only to take construction stresses occurring near unsupported edges. There are better ways to provide for these stresses, as explained at length in the writer's paper referred to previously.

The author's statement on page 1674* recommending concrete mixes not less rich than 1 : 2.5 : 5 is a little over-conservative, the writer believes, as most groined roofs have been built of 1 : 3 : 5 concrete, or of concrete of equivalent richness.

The author's statements as to the relative economy of the groined arch and reinforced concrete slab roofs are doubtless correct. The writer's estimates of relative cost, also contained in the paper referred to, showed that the reinforced concrete beam and slab roof costs about double that of the groined arch roof on works sufficiently large to permit repeated use of the forms.

* *Proceedings*, Am. Soc. C. E., October, 1922.

THE WATER POWER PROBLEM

A SYMPOSIUM

Discussion*

By Messrs. William T. Lyle, Howard R. Farnsworth, and J. W. Swaren.

William T. Lyle,† Assoc. M. Am. Soc. C. E. (by letter).‡—To the writer the leading thought in these presentations of different aspects of the Water Power Problem is expressed in the last word of the paper,§ by John P. Hogan, M. Am. Soc. C. E., namely, "co-operation". With it might be coupled two others, which he also uses, in order to bring out the nature of the problem and its bearing on the nation-wide problem of general internal improvements. These are "co-ordination" and "conservation", and to them the writer would add still another, namely, "control".

If Engineering is to be defined, as was Civil Engineering in the charter of the Institution of Civil Engineers, to wit, the "art of directing the great sources of power in Nature for the use and convenience of man as the means of production and traffic", this definition must be understood to possess a much broader significance than that attached to it heretofore. The modern engineer utilizes not only the powers and materials of Nature; he utilizes human, social, and legislative powers as well. He is no longer content to concern himself with technical matters alone; he must be more than an administrator of other men's determinations; he must be more than a counsellor and guide; he must possess the well-rounded aggressiveness of the successful business man as well. These qualifications, in the writer's opinion, are especially true of the water power engineer; the water power problem is of such a nature as to develop engineers of this high type. In the interests of National internal betterments, however, the future will probably bring to light engineers of still broader vision and still greater opportunity in the management of the mutually dependent material affairs of the nation.

Co-Operation.—Mr. Hogan has clearly shown the importance of a connected system of water powers whereby a needful and profitable interchange of power can be made between plants of different characteristics; he has pointed out the value of selective development, indicated the nature of a community of interest, and shown how available powers can best be utilized to secure a maximum economic return from each stream.

Co-operation in the water power field, however, is not enough. The problem should not be limited to the supply of existing demands; the demands also

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* Continued from January, 1923, *Proceedings*.

† Prof. of Civ. Eng., Washington and Lee Univ., Lexington, Va.

‡ Received by the Secretary, December 12th, 1922.

§ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1741.

may be created. Where power is available, it should be put to use. This result can be brought about through legislation and appropriation which bring together the producer and the consumer to the great advantage of both. No better way of accomplishing this can be found than in the construction of State and National highways through unimproved country. This applies especially to the Western States where also irrigation, a concomitant of water power development so often is necessary. The extension of the economic limits of high-voltage transmission make possible the opening of new districts regardless of their location with reference to the source of power.

Co-Ordination.—Power development, irrigation, flood control, water supply, canalization, and National parks are not separate considerations; they should be parts of one big problem and should be dealt with in their mutual dependence as interlocking interests. Independent operations in these several fields are not productive to the general public of a full return for the money expended.

Conservation.—The development of a water power not only secures the power while it can be had, but also operates to conserve the National supplies of coal and oil. As far as conservation is concerned, water power, water supply, and forest preserves may be grouped together. By conserving one resource, the others may be conserved also.

Control.—The harnessing of a stream for power production involves a beneficial regulation in the interests of flood control, irrigation, and of water supply farther down the valley. Among other advantages may be mentioned the stabilizing of channels, improved navigation, better sewer outlets, more advantageous conditions for bridges, elimination of ice gorges, and better heads during flood conditions for low-head power plants on the lower reaches of the stream.

Howard R. Farnsworth,* Assoc. M. Am. Soc. C. E. (by letter).†—The Act of Congress approved June 10th, 1920 (41 Stat., 1063), known as "The Federal Water Power Act" has had more than the usual allotment of troubles in its operation, beginning with the tardy date of its approval and continuing to include the realization that no adequate personnel had been provided in order to make the entire Act practically effective. In this latter respect, the situation appears not unlike that of the "Mineral Land Leasing Act" approved February 25th, 1920 (41 Stat., 437), a former Commissioner‡ of the General Land Office having predicted that no doubt the personnel problem would prove to be one of the weaknesses of the oil-leasing system. The Water Power Act, however, is very comprehensive in its terms and was conceived generally in sound judgment. It is to be regretted that our lawmakers did not give as careful attention to the legal and administrative features as did the engineers to the technical sections, which are excellent. There is much assurance to be found, however, in the fact that the Nation has as the active administrative heads of Federal water-power development and of reclamation by irrigation

* Member, Manual Board, General Land Office, Washington, D. C.

† Received by the Secretary, December 26th, 1922.

‡ "Oil Land Leasing Law of the United States", by Clay Tallman, *California Oil World*, Vol. XIII, No. 662 (2d Edition, May 26th, 1921), p. 78.

two engineers of outstanding ability, who are operating in their respective fields with a broad view looking to the early development of the water resources of the country.

In a recent description* of the newly invented 100-kw. triode vacuum tube, Mr. W. G. Housekeeper's important new method of making metal-to-glass seals is explained. Dr. Irving Langmuir who, by means of the construction of a series of five 20-kw. vacuum tubes, used in the place of alternators, recently transmitted radio messages across the ocean, has also discovered new processes, in conjunction with Mr. J. H. Payne, Jr., of the General Electric Company, and Dr. A. W. Hill, resulting in the development of a 1 000 000-watt (1 340 Eng. e. h. p.) vacuum tube which is intended primarily for the economical transmission of power over great distances. This super-tube which weighs only 60 lb., is described as a two-electrode tube, and it acts as a rectifier of current, and not as an oscillator, also depending, however, on the perfected process permitting the coalescence of glass and metal. Dr. E. F. W. Alexander, Chief Engineer of the Radio Corporation, is quoted in press dispatches as stating that these tubes will soon take the place of alternators in power transmission, and predicting that the transmission of electric power from Niagara Falls to New York City by means of these tubes is a possibility of the future. Dr. Langmuir also states that "it would be rash to predict the limitation of the ultimate use of vacuum tubes in the power field."†

It would be a curious comment if, by the time Congress has amended Section 2 of the Water Power Act so as to provide for adequate personnel in order to make it effective, engineering inventive genius has created the necessity also for the amendment of other sections of the Act in order to provide for new conditions, of possible development by science, in the hydraulic plant.

The preliminary factors of safety essential in every proposed water-power development include the following considerations:

1.—Feasibility of the project:

(*a*).—Physical: Proper physical conditions for the location of the power site; and,

(*b*).—Financial: Satisfactory market load centers at available transmission distances.

2.—Authority to proceed and control of the premises.

The subjects proposed in Section 1 of the foregoing considerations have been adequately covered by the others participating in this Symposium. The writer will discuss the questions raised in Section 2, which will immediately lead him to certain requirements of the Federal Water Power Act, and he will endeavor to set forth the principles, ignorance of which often causes those pitfalls sometimes encountered and which with prompt recognition may be avoided, that are involved in the "control of the premises", as they are construed by the Courts of last resort.

* By Dr. William Wilson, of the Western Electric Company, in the *Wireless Age* (New York); *Literary Digest*, Vol. 75, No. 8 (November 25th, 1922), p. 26.

† *New York Tribune; Literary Digest*, November 25th, 1922, p. 27.

Commerce Power

The Constitution.—Recall, for a moment, the period from May to September, 1787, when the Constitutional Convention sitting at Philadelphia, Pa., under the chairmanship of George Washington, was busily engaged in superseding the inadequate Articles of Confederation in favor of the "Virginia Plan" offered by Edmund Randolph, of that State, which plan, after much debate and many modifications, was finally adopted as the Written Constitution of the United States. One of the radical changes effected in this document was that delegating to Congress full power "to regulate commerce with foreign nations, and among the several States, and with the Indian tribes", which is to be found in Article 1, Section VIII, Clause 3, thereof. This clause forms the basis for the distinctions which are drawn in the ownership of the bed and the waters of navigable and non-navigable streams. The subject of commerce continues, after a period of 136 years, to be a paramount issue in Congress as is evidenced by the so-called "Ship Subsidy Bill" (H. R. No. 12817, 67th Congress, 3d Session), now pending and in debate.

In Colonial times, the English Unwritten Constitution, common-law practice, and usage had prevailed in America, and judicial decisions were based exclusively on precedents established in English Courts. The State Constitutions of the United States continue to be construed in the light of this common law, inasmuch as the common law is regarded as having been in existence prior to the Constitutions and as remaining afterward, limited only by such restrictions as the Constitutions impose.*

When, in 1789, the new republic was organized it became necessary to modify the existing principles of jurisprudence and to adapt them to the altered theory of government. The confirmation of John Marshall as Chief Justice of the United States, in 1801, marked an epoch in the history of the Nation, inasmuch as he immediately proceeded to establish the structure of American law on a firm and enduring basis, and for 35 years he bequeathed to the future a great number of valuable decisions, in which the principles of American jurisprudence are set forth with invincible logic and unvarying clearness.

Robert Fulton made his trip up the Hudson River from New York City to Albany, N. Y., in the *Clermont,* in 1807, and, for many years thereafter this first steamer continued to ply the Hudson, thus revolutionizing the old methods of river navigation. The Legislature of the State of New York had granted an exclusive right to Robert Livingston and to Fulton to navigate the waters of that State "by fire or steam", in consideration of their services in making the steamboat practicable, and in Gibbons *v.* Ogden (9 Wheaton, 1), in 1824, Chief Justice Marshall delivered the opinion of the Court in one of the first pronouncements of the restrictions imposed on State authority, defining therein the power exercised by Congress over interstate commerce and navigation. This decision continues to serve as a standard in the consideration of the questions raised.†

* State *v.* Noble, 118 Ind. 350; Mattox *v.* United States, 156, U. S. 237.

† See, also, Pensacola Tel. Co. *v.* W. U. Tel. Co., 96 U. S. 1; Texas & Pac. R. R. Co. *v.* Interstate Commerce Comm., 162 U. S. 197.

The Relation to Water-Power Development.—In the construction of water-power projects, the dams, head-races, diversion structures, other appurtenant works, and the primary reservoirs, will invade the bed, the banks, and the waters of the stream.

Section (4), Sub-Section (*e*) of the Water Power Act provides, in part, that before the issuance of a preliminary permit to an applicant for a license, the State authorities shall be advised of the proposed action, and publication thereof shall be broadcasted, locally, for a period of eight weeks. Section (5) of the Act states that the sole function of a preliminary permit is to secure priority to the claimant in his application for a license, and to provide for the necessary examinations, surveys, maps, plans, etc., in order to set forth, clearly, all the conditions which obtain. Section (9), Sub-Section (*b*), of the Act requires the applicant for a license to submit satisfactory evidence so as to show proper compliance with the laws of the State, with respect to the "bed and banks and to the appropriation, diversion and use of water for power purposes," etc. When the power enters into interstate or foreign commerce a certain regulation thereof is stated, in Section (20), to fall within the jurisdiction of the Interstate Committee under the procedure outlined in the Act approved February 4th, 1887, as amended (24 Stat., 379).*

It is thus indicated that irrespective of whether the stream is navigable or non-navigable, it has always been necessary for the promoter of a water-power development to proceed in accordance with particular Federal and State laws which apply according to the physical conditions that obtain.†

Factors Determining Navigability

O. C. Merrill,‡ M. Am. Soc. C. E., has cited the cases of the Saco, the Connecticut, and the Menominee Rivers in the New England and the North Central States which, although determined on investigation to be technically navigable, were held not to be navigable waters within the definition of the Act, in virtue of the full protection afforded the existing interstate commerce by the laws of the particular States. Federal licenses accordingly were not required for power installations on these streams.

It is fully recognized that the statement of navigability‡, as set forth in Section (3) of the Water Power Act, regulates the administration of the provisions thereof and that, under Section (23), if the Commission shall find that substantial interests of interstate or foreign commerce are not affected by the proposed construction of a dam or other obstruction across a stream, a Federal license to proceed is not required. This fact, however, should not be confused with the classification and ownership of the public or the State

* See, also, for this subject, State Freight Tax, 15 Wall. 232.

† For State control over domestic commerce, see Welton *v.* Missouri, 91 U. S. 275; Veazie *v.* Moor, 14 How., 568–574; Cooley *v.* Wardens, 12 How., 299. The police power of States over either domestic or interstate commerce is explained in R. R. Co. *v.* Husen, 95 U. S. 465; Chi Lung *v.* Freeman, 92 U. S., 275; State Freight Tax, 15 Wall., 232; Brown *v.* Houston, 114 U. S. 622; Lake Shore Ry. Co. *v.* Ohio, 173 U. S. 285; License Cases, 5 How., 504; Munn *v.* Illinois, 94 U. S. 113; Commonwealth *v.* Alger, 7 Cush. (Mass.) 53; Lawton *v.* Steele, 152 U. S. 133.

‡ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1762.

lands involved, in any of the public land States,* which rest on the similar conception of navigability, that which is conveyed in the following definition:†

"The settled rule in this country that navigability in fact is the test of navigability in law, and that whether a river is navigable in fact is to be determined by inquiring whether it is used, or is susceptible of being used, in its natural and ordinary condition as a highway for commerce, over which trade and travel are or may be conducted in the customary modes of trade and travel on water."

This definition is distinguished in all the cases cited and is enlarged upon in detail especially in the first two. The quotation itself is taken from the decision of the U. S. Supreme Court in Oklahoma *v.* Texas, United States Intervener, No. 20 Original, delivered on May 1st, 1922. The decision is very comprehensive and contemplates all phases of "navigability in fact", ownership of the bed and banks and of riparian rights. It is not yet published in the volumes of U. S. Reports, being found only in the advance sheets thereof, the "Lawyer's Editions", and also in pamphlet form. All the numerous excerpts, hereinafter quoted up to the Section headed, "Meander Surveys and High-Water Level", will be understood to have been taken from this decision, or, where a specific reference to it is made again, the form "66 L. Ed. 444" will be used. The river under consideration in this decision is the Red River, and the details of the suit with photographs of the stream are exhibited in a paper by the writer, entitled "A Review of Important Developments in the Science of Cadastral Resurveys as Executed by the United States Government, with Ethical Discussion Thereof."‡

Legal Inference.—Legal inference of navigability, which is sometimes alleged to arise from the action of surveying officers in closing official surveys on the meandered lines of a stream, is held to have "little significance".§ A similar inference is not attached to those rivers over which Congress, in permitting the construction of bridges, has added the precautionary provision that there should be no interference with navigation.‖ In like manner, those statements taken from early publications and repeated in later ones, which announce the fact of the navigability of a stream, are not sufficient to establish a legal inference:¶

"These statements originated at a time when there were no reliable data on the subject, and were subsequently accepted and repeated without much concern for their accuracy. Of course they and their repetition must yield to the actual situation as developed in recent years."

Decisions of State Courts are also sometimes referred to as determining that a river is navigable in fact: "The United States was not a party and is

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* For public land States, see, *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 573.

† The Daniel Ball, 10 Wall. 557, 563; The Montello, 20 Wall. 430, 439; United States *v.* Rio Grande Co., 174 U. S. 690, 698; United States *v.* Cress, 243 U. S. 316, 323; Economy Light & Power Co. *v.* United States, 256 U. S. 113, 121; McGilvra *v.* Ross, 215 U. S. 70, 78; McManus *v.* Carmichael, 3 Iowa 1.

‡ *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 546.

§ Also, see, Barden *v.* N. P. R. R. Co., 154 U. S. 288, 320; Gauthier *v.* Morrison, 232 U. S. 452, 458; Harrison *v.* Fite 148 Fed. 781, 784.

‖ 66 L. Ed. 444.

¶ Also, see, Missouri *v.* Kentucky, 11 Wall., 395, 410.

not bound.* There is in the opinion no statement of the evidence, so the decision hardly can be regarded as persuasive here."

The characteristics of a stream are frequently represented so as to establish that the transportation has been and must be exceptional, and that it is confined to the irregular and short periods of temporary high water: "A greater capacity for practical and beneficial use in commerce is essential to establish navigability."†

Control of the Premises

Navigable Waters.—Although the United States has the power and authority to grant, for appropriate purposes, rights and titles below the high-water mark along and under navigable waters in the Territories,‡ it is its policy not to do so, but rather to leave the bed and waters of navigable streams free from any easement of the upland proprietor.§ In virtue of the constitutional rule of equality among the States, each new State becomes, on the date of its admission into the Union, the owner of the navigable waters within its boundaries and of the land underlying the same,‖ an ownership which was reserved and retained by each one of the original States. The character of the State's ownership in the land and in the waters of navigable streams is the full proprietary right, subject, however, to the primary control thereof by the United States over interstate commerce and navigation. Whether a conveyance by any State, of land abutting on navigable streams, confers on the grantee any right or interest in those waters, or in the land under the same, is a matter wholly of local law.¶ On such questions, the provisions of the Constitution and Statutes of the particular State involved and the decisions of its highest Court are regarded as conclusive.**

Thus, a State has title to the soil below ordinary high-water mark along navigable streams,†† and the shore is considered rather a part of the water than as land. The line marking the true mean high-water elevation of a stream is determined from the river bed, and that only is river bed which the river occupies long enough to wrest it from vegetation (Houghton *v.* Railway Company (47 Iowa 370)). A bank is defined as the continuous margin where vegetation ceases, and the shore is designated as the sandy space between it and low-water mark (McCullough *v.* Wainwright (14 Penn. St., 59)). The laws of each State, therefore, must be investigated to determine the ownership of the soil below mean high-water level along navigable streams.

* Also, see, Economy Light & Power Co. *v.* United States, 256 U. S. 113, 123.

† Also, see, United States *v.* Rio Grande Co., 174 U. S. 690, 698–699; Leovy *v.* United States, 177 U. S. 621; Toledo Liberal Shooting Club *v.* Erie Shooting Club, 90 Fed. 680, 682; Harrison *v.* Fite, 148 Fed. 781, 784; N. American Dredging Co. *v.* Mintzer, 245 Fed. 297, 300.

‡ Brewer-Elliott *v.* United States, 270 Fed. 100; Irvine *v.* Marshall, 20 How. 561; United States *v.* Winans, 198 U. S. 371; Prosser *v.* N. P. R. R., 152 U. S. 59.

§ McGilvra *v.* Ross, 215 U. S. 70–79; Joy *v.* St. Louis, 201 U. S. 332; Scranton *v.* Wheeler, 179 U. S. 141, 190; Kansas *v.* Colorado, 206 U. S. 46, 93.

‖ 66 L. Ed. 444; Scott *v.* Lattig, 227 U. S. 229, 242–243; Pollards Lessee *v.* Hagan, 3 How. 213; Weber *v.* Board of Harbor Commissioners, 18 Wall. 57, and cases cited.

¶ Shively *v.* Bowlby, 152 U. S. 1, 48–58; United States *v.* Mission Rock Co., 189 U. S. 391; U. S. Supreme Court in Port of Seattle *v.* O. & W. R. R. Co., January 31st, 1921.

** St. Anthony Falls Water Power Co. *v.* St. Paul Commissioners, 168 U. S. 349.

†† Goodtitle *v.* Kibbe, 9 How. 471.

There is no uniformity of State laws on this subject. Either of three different rules will be found to obtain, namely, that the riparian owner's title extends to, (1) mean high-water mark; (2) low-water mark; and (3) the middle thread of the current. For example, Section 4529 R. C. of the State of Montana, 1907, provides that:

"Except where the grant under which the land is held indicates a different intent, the owner of the land when it borders upon a navigable lake or stream, takes to the edge of the lake or stream at low-water mark * * *."

On February 25th, 1895, in Gibson *v.* Kelley (15 Montana 417, 422), the State Supreme Court held that rule always to have been the order in Montana. In the State of Michigan, the riparian owner's title is declared to extend to the middle thread of the current of the stream, irrespective of whether it is navigable or non-navigable.*

In pursuance of the Montana rule, it would appear that in this and in other public land States where similar laws obtain as to the ownership of the shores along navigable streams, and where the United States becomes a riparian owner to low-water mark, the right of the Government to survey and to dispose of lands up to low-water mark would suggest itself, acting as a trustee for its future grantee. The United States, however, has wisely abstained from extending (if it could extend) its surveys and grants beyond the limits of ordinary high water (Barney *v.* Keokuk (94 U. S. 388) and Frank Burns (10 L. D., 365)).

It is also believed that there is no instance in which the United States after having disposed of the land to ordinary high-water mark has departed from the long-established rule of common law and, afterward, asserted a right to dispose of the space between that and low-water mark (The Mayor of Mobile *v.* Eslava (9 Porter, 578)).

Non-Navigable Waters.—In the case where the United States owns the bed of a non-navigable stream, in disposing of the upland along one or both of its banks, it is, of course, free when disposing of the upland to retain all or any part of the river bed; and, in any particular instance, the question of whether it has done so is essentially a matter of what it intended in the disposal.† Where a contrary intention is not shown by treaty, statute, or by the terms of its patent, the United States "will be taken to have assented that its conveyance should be construed and given effect, in this particular, according to the law of the State in which the land lies."‡

John P. Hogan, M. Am. Soc. C. E.,§ has described the difficulties, formerly encountered in the State of New York, in the development of harmonious schemes of power development and operation, on account of the necessity of

* Butler *v.* G. R. & Ind. R. R. Co., 85 Mich., 246. Also, see, comment on this holding in 159 U. S. 87, 91, 96, relative to the lack of a prior adjudication by the U. S. Land Department in respect of the islands involved.

† Wilcox *v.* Jackson, 13 Pet., 498, 516–517; Irvine *v.* Marshall, 20 How. 558; Gibson *v.* Chouteau, 13 Wall. 92, 99; Utah Power & Light Co. *v.* United States, 243 U. S. 389, 404; Kean *v.* Calumet Canal Co., 190 U. S. 452, 460.

‡ See, also, Hardin *v.* Jordan, 140 U. S. 371, 384; Mitchell *v.* Smale, 140 U. S. 406, 413–414; G. R. & Ind. R. R. Co. *v.* Butler, 159 U. S. 87, 92; Hardin *v.* Shedd, 190 U. S. 508, 519; Whitaker *v.* McBride, 197 U. S. 510, 512, 515–516; and, see, Railroad Co. *v.* Schurmeir, 7 Wall, 272, 287, *et seq.*

§ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1755.

first obtaining unanimous agreements of private riparian owners relative to the interrupted flow of the streams. He then explains the organization of the River Regulating District which is based on a mutual understanding among all the power owners along a stream.

The common law recognizes certain rights of riparian proprietors in the natural flow of a stream, and it extends riparian ownership along the banks of non-navigable waters to the middle of the stream.* In this regard, the common-law rule, although modified somewhat, will be found to prevail in most of the States, and ordinarily a riparian proprietor along non-navigable rivers will take to the *filum aquæ,* or the center thread of the stream.

The terms, "middle of the main channel" and "mid-channel," are often used in defining the limit of ownership. When applied to navigable streams such terms usually refer to the thread of the navigable current, and if there are several, to the thread of the one best suited and ordinarily used for navigation.†

In non-navigable streams, however, which are often without a channel of any permanence or a continuous or dependable flow, "the channel extending from one cut-bank to the other, which carries the water in times of substantial flow * * * was the only real channel and therefore is the main channel."‡

Riparian Rights.—The proprietor of lands bordering along the banks of streams also acquires a riparian title to land that attaches thereto by imperceptible degrees as accretions, irrespective of whether the stream is navigable or non-navigable, or whether the additional frontage is the result of the deposit of suspended soil or of the gradual recession of the stream. In preparing the plat of an original official survey from the field notes, the courses of the meander survey are represented thereon as the border line of the stream. It is thus shown to a demonstration, in so far as the original meander survey restricts the title of the grantees of the United States along navigable streams, that the watercourse is the boundary, and not the line of the meander survey at the border of the upland.§ Where, by action of the water, a river bed has gradually changed, the ordinary high-water mark also changes and the ownership of adjoining land follows with it.‖

Portions of the bank of a stream will often be swept away in times of flood after the original survey of the meander lines and after the identification of the surveyed units of disposal abutting them. Some of these surveyed tracts will become thereafter a part of the river bed and others formerly non-riparian will become riparian. In the case of non-navigable streams, where the law of the State confers a riparian ownership to the center thread of the stream, even if tracts which formerly were riparian upland had thus become river bed before their disposal, the proprietorship under the disposal thereof would reach to the middle of the stream, providing a prior disposal

* Middleton *v.* Pritchard, 3 Scammon, 510.

† Iowa *v.* Illinois, 147 U. S. 1; Okla. *v.* Texas, 66 L. Ed. 444.

‡ Oklahoma *v.* Texas, etc., 66 L. Ed. 444.

§ R. R. Co. *v.* Schurmeir, 7 Wall, 272, 286–287; Minto *v.* Delaney, 7 Oregon, 337, 342.

‖ Minto *v.* Delaney, 7 Oregon 337, 343; 11 Ohio 314; Steele *v.* Sanchez, 33 NW. Rep. 367; Lockwood *v.* R. R. Co., 37 Conn. 387; Grant *v.* Fletcher, 283 Fed. 245.

had not already been made of the adjacent upland then actually riparian. When, however, the disposal of the tracts thus found to be situated in the river bed is made subsequent to that of the sub-divisions adjoining on the upland back of them, theretofore non-riparian, but which had become riparian, then the conveyance of these latter tracts, as a rule, will include the ownership to the center line of the stream.*

Meander Surveys and High-Water Level

"Meander lines will not be established at the segregation line between dry and swamp or overflowed land, but at the ordinary high-water mark of the actual margin of the river or lake on which such swamp or overflowed lands border."†

Ordinarily, all swamp and overflowed lands have been patented to the State under the swamp-land laws (Sec. 2480, U. S. R. S., Act of September 28th, 1850 (9 Stat., 519)). These lands are clearly defined in Heath *v.* Wallace (138 U. S. 573, 584). Several varieties of native forest trees are found only within the zone of swamp or overflowed lands. However, all timber growth normally ceased at the margin of permanent water.

"Mean high-water elevation will be found at the margin of the area occupied by the water for the greater portion of each average year; at this level a definite escarpment in the soil will generally be traceable at the top of which is the true position for the surveyor to run the meander line."‡

It is not practicable, however, in public-land surveys to meander a stream in such a manner as to reproduce all the minute windings of the true mean high-water elevation, and, in practice, only the general courses and distances of these sinuosities are followed.

Correction of Erroneous Meander Lines.—When the United States has disposed of the lands abutting the meander lines of streams, without reservation in the conveyance, it, as a rule, retains no further jurisdiction over the stream area, irrespective of whether it is navigable or non-navigable in character, provided, however, that the original survey of the meander line was reasonably correctly established.

The Land Department of the Government is charged with the duty of primarily determining what are public lands subject to survey and disposal under the public land laws.§ It also has the authority to correct erroneous or fraudulent surveys wherever it is proved to its satisfaction that public lands which should have been surveyed, have been left unsurveyed, either as a result of incompetence, inadvertence, mistake, or fraud.‖

* Oklahoma *v.* Texas, etc., 66 L. Ed. 444.

† "Manual of Instructions for the Survey of the Public Lands of the United States," Gen. Land Office, 1894, p. 57, and repeated in all subsequent editions.

‡ Advance sheets of a revision of the "Manual of Instructions for the Survey of the Public Lands of the United States," 1919, p. 213.

§ Sections 453 and 2476, U. S. R. S.; Kirwan *v.* Murphy, 189 U. S. 35; Brown *v.* Hitchcock, 173 U. S. 473.

‖ Omagin *v.* Powell, 126 U. S. 691; Horne *v.* Smith, 159 U. S. 40; French Glenn Livestock Co. *v.* Springer, 185 U. S. 47; Miles *v.* Cedar Point Club, 175 U. S. 300; Tubbs *v.* Wilholt, 138 U. S. 261; C. & D. Lumber Co. *v.* St. Francis Levee District, 232 U. S. 186; Gauthier *v.* Morrison, 232 U. S. 452; Producers Oil Co. *v.* Hansen, 238 U. S. 325; Lee Wilson & Co. *v.* United States, 214 Fed. 630; 227 Fed. 827; affirmed by U. S. Supreme Court, November 5th, 1917.

The Secretary of the Interior, however, in the case of The Marshall Dental Manufacturing Company (32 L. D., 553), and in numerous other departmental land decisions, has announced the policy of the Interior Department to be as follows:*

"The Department has the power to correct surveys upon a proper showing, but, as has frequently been said, the proper rule is to refuse to disturb the public surveys except upon the clearest proof of accident, fraud, or mistake, where a resurvey may affect the rights or claims of any one resting upon the original survey."

Although the long-established rule, to the effect that the actual physical corners and monuments of an original survey hold precedence over the positions thereof as indicated in the record field notes, is quite general in its application, an instance will occasionally arise where an absurdity or a fraud is developed in the strict application of such a principle; in which case, the courses and distances of the original record may hold over the original monuments.† If such a case should arise or other erroneous conditions are developed in the identification of a false original meander survey, the meander lines may then be held to have become a strict boundary. If one should desire to investigate the degree of accuracy of the early meander surveys along any stream, for the determination of a possible irrelation when compared with a true mean high-water elevation which might clearly have existed, as such, at the time of the original survey, the method of traverse adjustment will be utilized for this comparison, such as is explained by the writer in his paper, previously mentioned, covering the practice of the Cadastral Engineering Service of the General Land Office.‡

The Survey of Islands.—Artificial lakes and reservoirs, as a rule, are not segregated from the public lands, however, in the original survey of the mainland fronting on any non-navigable body of water, all islands opposite thereto above mean high-water elevation are subject to survey and disposal.

"Even though the United States may have parted with its title to the adjoining mainland, an island in any meandered body of water, navigable or non-navigable, known or proven to have been in existence above the mean high-water elevation at the date of the admission of a State into the Union, and at the date of the survey of the mainland, if omitted from said original survey, remains public land of the United States, and, as such, the island is subject to survey."§

If, however, an island is formed on the bed of a navigable river subsequent to the date of the admission of the State into the Union, the title to such island is vested in the State and not in the United States.‖

* This policy was also applied on October 30th, 1914, in a ruling on an application for survey by the Klickitat White Pine Co. (Duluth 011106).

† Security Land & Exploration Co. *v.* Burns, 193 U. S. 167–179; Ainsa *v.* United States, 161 U. S. 208, 229; White *et al.* *v.* Luning, 93 U. S. 514–524; Shipp *et al.* *v.* Miller's Heirs, 2 Wheat. 316.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), pp. 544, 545, Figs. 9 and 10.

§ Advance sheets of a revision of the "Manual of Instructions for the Survey of the Public Lands of the United States", 1919, p. 217. See, also, McManus *v.* Carmichael, 3 Iowa 1; Wiggenhorn *v.* Kountz, 8 Amer. State Rep. 150; Hardin *v.* Minn. No. Ry. Co., 84 Fed. 287; in which it was shown that the Government had treated the islands involved as separate property.

‖ Pollard *v.* Hagan, 3 How. 212; Widdicombe *v.* Hosemiller, 118 Fed. 293; Hardin *v.* Jordan, 140 U. S. 371.

Extension surveys, as described in the foregoing, which are executed for the purpose of embracing public lands along bodies of water held to have been erroneously omitted from the original survey, are generally not undertaken by the General Land Office until after the receipt of a formal application therefor, a careful examination of the land in question, and proper authorization by the Secretary of the Interior to proceed.

Consequences of Indiscretion

It is not the purpose of this discussion to alarm the promoter or the owner of any water-power project, inasmuch as all the laws which relate thereto are administered with the broad view of encouraging such developments. It is, however, the part of wisdom to know the premises involved and to establish, definitely, the control thereof before proceeding with the expenditure of large sums of money in project construction, always remembering, where the case may apply, that a naked, unlawful trespass cannot initiate a right to any part of the public domain,* and that "possession not based on a legal right, but secured by violence and maintained by force and arms, cannot furnish the basis of a right enforceable in law."†

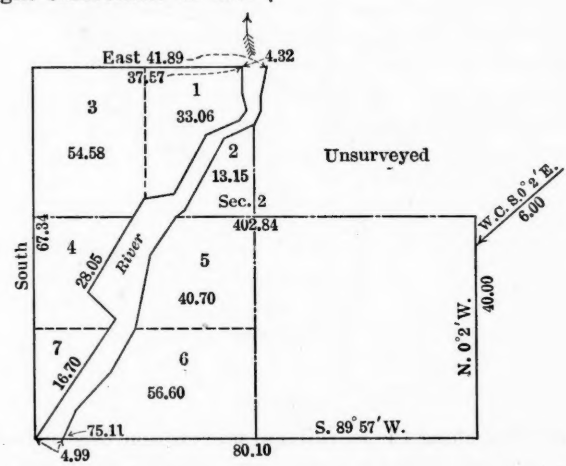

Fig. 26.

With the foregoing principles in mind, attention is invited to Figs. 26 and 27, wherein is portrayed the situation now confronting a certain private company which owns a completely installed power unit of some magnitude, one which is locally well known. The physical conditions exhibited in Fig. 27 are identical with those which are represented as existing. All names, however, and other means of identification are fictitious and are purposely so on account of the pendency of the case.

This power installation was completed in 1907, whereas the control of the premises was not adequately considered until 1922. The company holds a fee-

* Swanson *v.* Sears, 224 U. S. 182.

† Lyle *v.* Patterson, Syllabus, 228 U. S. 215.

simple patent for Lots 1, 2, 3, 4, and 5, Section 2, in the position as shown on the plat of the original survey, Fig. 26. The dam has been placed in the natural position for it at the outlet of a natural basin and at the entrance to a narrow canyon which is formed by the river. In this position, however, the east abutment may possibly be found to rest on extant public lands of the United States, which are subject to survey and disposal under the public land laws.

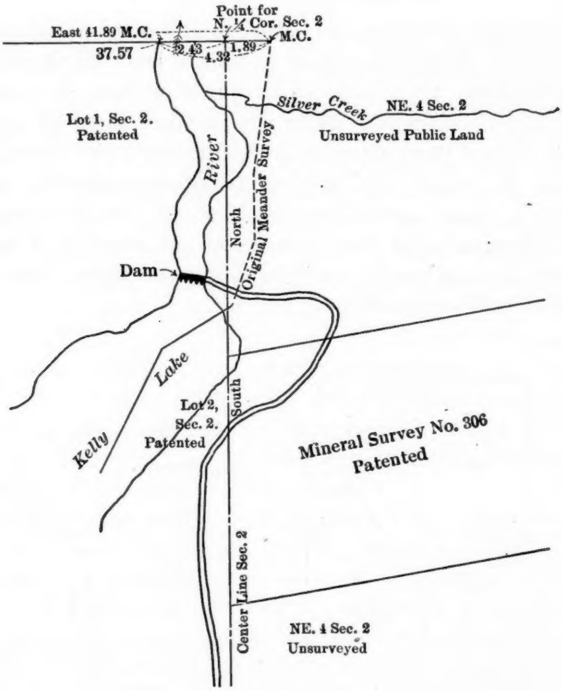

FIG. 27.

If the land lying along the meridional mid-section line of Section 2, inside the erroneous original meander survey, and between the north angle-point of the original survey of Lot 2 (see Fig. 27) and the true mean high-water level of the river, is determined to be vacant public land of the United States, then the only apparent possibility that the east abutment of the dam might rest on the company's property, would lie in the circumstance that the exterior side line of the riparian addition to Lot 2 which extends westerly to the center of the stream, might fortuitously intersect the east bank at a point north of the dam.

If the company has not already the necessary ownership, it may acquire the title through either a special Act of Congress, or by proceeding under the various land laws, or, if an application for a license is filed, under Sections 21, 23, and 24 of the Federal Water Power Act, immediate protection will doubtless be obtained. It is emphasized, however, that the initiative in

all these actions rests with the power company. In this particular case, the past record of apparent indifference to security does not indicate a high degree of understanding of such matters, or of initiation. Instances of similar imprudence may exist elsewhere, which are not realized by the parties most in interest. These cases may develop into situations of much greater complexity, with reference to the inception of private adverse rights, before they receive even ordinary consideration.

A fully developed and secured water-power plan is the greatest of all measures for the conservation of natural resources, inasmuch as perpetual benefits are made available thereby. The enormous quantities of coal, fuel oil, and all other forms of fuel now being expended in the generation of steam, may be saved by the hydro-electric power developed, irrigation of agricultural lands is made possible by the use of the water leaving the power wheel, and the possibility of destructive floods is reduced through the storage and control of water in the reservoirs.

The potential water power of all the streams of the United States is not a matter of accurate knowledge. Total amounts will vary widely according to the particular basic assumptions of each estimate. The most comprehensive survey of both the developed and the undeveloped water-power resources of this country is that accomplished by the Government, and which is to be found in the Report of the Secretary of Agriculture dated January 20th, 1916, and published as Senate Document No. 316, 64th Congress, 1st Session. This report is the result not only of independent corrective investigations made in 1915, but also of studies and additions accomplished by several Federal agencies.*

From page 17 of the report made in 1916 by the U. S. Forest Service, one finds that the total undeveloped power resources of the streams of the United States is conservatively estimated at a minimum of 27 943 000 h. p., and a maximum of 53 905 000 h. p. The minimum amount represents continuous power at periods of lowest flow, and the maximum, the average power available during one-half the time. The use of storage was not considered in these estimates, and the efficiency of the water-wheel was assumed at 75 per cent.

In contrast with this amount, an estimate based upon other assumptions is that repeated by Mansfield Merriman,† M. Am. Soc. C. E., which states that the total potential water power of the United States is 200 000 000 h. p. Mr. Hogan‡ estimates that 2 000 000 continuous h. p. represents one-half the undeveloped potential energy of the Niagara River; various other engineers consider that 4 000 000 h. p. may be developed at an economic cost in the State of California alone, and M. M. O'Shaughnessy,§ M. Am. Soc. C. E., believes that a close estimate of all the possibilities of that State lies between 8 000 000 and 9 000 000 h. p.

* Report, in 1908, of the Bureau of the Census to the National Conservation Commission (Senate Doc. No. 676, 60th Cong., 2d Session); report, in 1908, of the U. S. Geological Survey, also submitted to the Conservation Commission (*Water Supply Paper No. 234*); report of the Commissioner of Corporations, dated March 4th, 1912; and the report, in 1916, of the U. S. Forest Service (Senate Doc. No. 316, *supra*).

† "Treatise on Hydraulics," 8th Ed. (1907), p. 371.

‡ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1741.

§ *Proceedings*, Am. Soc. C. E., December, 1922, p. 1881.

A statement* has been made that, in one of the large steam plants of New York City, each horse-power developed now costs about $50 per operating year. If this estimate is only approximately correct, it is fully realized that the development of the power resources of our streams is the big problem, the answer to which represents one of the most impending needs of this country. The enactment of the Federal Water Power Act is a long step toward the solution of this problem.

J. W. SWAREN,† M. AM. SOC. C. E. (by letter).‡—These papers cover every phase of the water-power problem, except the obtaining of funds and the relation of the investing public to programs of extension.

In his paper on the "The Prospective Competitor Method of Valuation of Property",§ M. L. Byers, M. Am. Soc. C. E., discussed certain features of the rights of existing investors. Of equal or greater import are the rights of investors in new projects. They should know that the market is large enough to absorb the new unit which their money is developing and that the future growth of revenue-producing load will not be outstripped by increased competitive development, or, perhaps, completely dissipated before the net revenues wipe out the early operating deficits.

Rate-making bodies are not inclined to look with favor on rates sufficiently high to re-absorb such deficits. Yet such rates are as real as interest on bonds during the construction period and, on the basis of present worth, as amenable to capitalization.

On the other hand, programs on scales of insufficient size are equally likely to financial disaster. Recently, the writer has observed a semi-industrial power station of the usual efficiency, which has, as its only outlet, sale of power to an existing network with large station capacity. The maximum block of power which the new station can contract, can only be marketed at a price lower than operating expenses. As a result, more than one-half the station is idle, whereas a station of double the size would have a market, and the distributing organization would not be under the necessity of increasing its generating capacity, as it now is.

O. C. Merrill,‖ M. Am. Soc. C. E., mentions that certain activities are not undertaken because of lack of funds and insufficiency of personnel, more particularly appraisal work. The writer would like to draw his attention to a rapid method of appraisal that is proving successful where appraisals are necessary in determining income taxes. The books of the company are taken as a basis—costs being checked by auditors—a brief physical inspection is made by an engineer to determine the general conditions of the original construction and of present-day replacement methods. Factors based on general economic studies are applied to determine present-day physical values. An economic study is then made of the particular company, and curves of ex-

* Dr. Edwin E. Slosson, of Science Service, in its *Daily Science News Bulletin* (Washington, D. C.); *Literary Digest*, Vol. 75, No. 9 (December 2d, 1922), p. 25.

† Engr., Bureau of Internal Revenue, Washington, D. C.

‡ Received by the Secretary, January 8th, 1923.

§ *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1313.

‖ *Proceedings*, Am. Soc. C. E., November, 1922, p. 1758.

pectancy are developed. Net profits actually received in subsequent years are proving that accuracy is being obtained by these methods of economic valuations.

Amplifications of Kelvin's law have been used with success by the writer for economic predeterminations. Its application to the solution of general engineering economics offers little more complication that its application to the economic determination of a copper network.

The investor is entitled to a careful study of the market which is to be served, as well as how it is to be served, preferably by a disinterested investigator, with a continuing experience of wide scope. The writer hopes that Mr. Merrill had such studies in mind when he spoke of important, but not yet undertaken, duties. He states that the Federal Commission is concerned largely with the enforcement of a contract obligation. Protection of the general public, of which he speaks, requires that the contractor be capable of fulfilling his contract throughout its life. The necessary investigations to determine those features already assumed by the Commission should supply most, if not all, the data to enable a satisfactory determination of expectancy.

Water-power securities should be sound under all conditions of general economic stress. Soundness is only possible when the market is assured for the life of the security at the least. This can be predetermined by engineering methods, and should be checked as carefully as any other part of the design.

THE DESIGN OF STRUCTURAL SUPPORTS FOR TURBO-GENERATORS

Discussion*

By Messrs. W. E. Belcher, Austin H. Reeves, F. A. Ericson, James H. Richardson, and F. M. Beer.

W. E. Belcher,† M. Am. Soc. C. E.—The development of the large steam turbine generator unit marks a rapid advance in the science of Mechanical Engineering, which has made necessary the special structure discussed by the author. It is to-day an appropriate topic for consideration, although ten years ago it was in its infancy, and ten years hence may have lost all practical interest.

At the World's Fair in 1904, one's attention was attracted by a loud shrieking noise which came from a steam turbine forming part of the exhibit of the Westinghouse Company. This exhibit was perhaps the introduction of the steam turbine to American engineers. For a number of years thereafter, the adoption of the turbine was very slow, the usual form being the vertical unit with capacities of 2 000 to 5 000 kw. These units were mounted on circular or octagonal concrete piers that were hollow in the center for access to the step bearings. This form was virtually confined to the impulse type of blades, and did not lend itself to expansion in the capacity of the unit.

About 1914, the manufacture of the vertical turbo-generators was discontinued, and during the last ten years, the horizontal type has rapidly been replacing all other forms of electric power generators in central station practice.

In the speaker's experience, single units with capacities as large as 35 000 kw. have been mounted on frames of the type described in the paper. A unit of 60 000 kw. has been installed at Cheswick, Pa., in which case, however, there were three separate turbines, one high pressure and two low pressure, each with a separate support. In designing these supports, it is the speaker's practice to use the following maximum unit stresses in structural steel members:

* This discussion (of the paper by Edward H. Cameron, Assoc. M. Am. Soc. C. E., presented at the meeting of December 20th, 1922, and published in January, 1923, *Proceedings*), is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

† Structural Engr., Dwight P. Robinson & Co., New York City.

Compression on columns........	$8\,000 - 45 \frac{L}{R}$	lb. per sq. in.
Bending on extreme fiber of net section	8 000 lb.	" " "
Rivet values, shop-driven, in shear	6 000 lb.	" " "
Rivet values, shop-driven, in bearing	12 000 lb.	" " "
Field driven rivets............	four-fifths of the values given	

Good judgment is more important, however, than fixed unit stresses, and the quantity of steel and concrete used should be as great as is reasonably justified rather than as small as possible. In erecting the machine, the top of the rough concrete slab and the steel frame are left about 1½ in. below the base casting, which is grouted in solid after all work has been aligned and leveled. For this reason, the entire deflection is taken care of by the grouting, so that the speaker does not consider deflection to be the criterion.

The speaker believes that provision should be made for the full weight of the condenser where the connection between condenser and exhaust nozzle is bolted solidly, even if springs are placed under the condenser.

Co-operation between the mechanical and structural engineers is essential in the early stages of the design, and a slight change in the mechanical layout of the auxiliary equipment may permit the placing of a diagonal or the deepening of a girder, thus improving considerably the structural design.

The cramped quarters for equipment beneath the unit, including the condenser, pumps and piping, generator, air inlet and discharge, electrical connections, etc., make a design of reinforcing concrete especially difficult. If the severe limitation of space as it develops makes one of the members out of proportion to the remainder, it is likely to cause trouble. As a result of these limitations, steel supports are generally more satisfactory. The platform floor is usually of concrete and the supporting girders are filled with concrete, both for structural bracing and for the mass resistance to vibration.

Every effort should be made to have the main longitudinal girders continuous for the full length of the support. The longitudinal bracing is usually placed under the electrical end of the machine, but transverse bracing must be used at each transverse bent. This bracing should be in the form of diagonals and struts, as far as possible, rather than in the form of brackets and column flexure. The entire structure should be separated from the surrounding floors, and platforms or walkways supported by slender hangers or light cantilever brackets should be avoided on account of vibration.

It is the speaker's practice to assume, for the design of the bracing, a horizontal load in any direction acting at the level of the main platform, equal in amount to one-fifth the total weight of the turbo-generator. Such a design based on the low stresses mentioned will give a substantial system of bracing in each direction.

Even if a structurally perfect design has been secured, experience has shown that local conditions may cause trouble. There are cases recorded in which one column stood in direct sunlight while others were in shadow, thus

causing a distortion of the frame and the destruction of the generator. In another case, distortion was caused by a cold draft at one end of the frame and considerable expense was entailed until the cause of the trouble was found. It is essential, therefore, to keep the frame at a fairly uniform temperature, and, with this in view, the steel members are sometimes encased in concrete, as noted by the author.

AUSTIN H. REEVES,* ASSOC. M. AM. SOC. C. E.—The main difference of opinion between Mr. Cameron and the speaker is in regard to the use of the "footing" or "mat". The speaker believes:

(*a*) That "separate footings" should never be "used for the individual columns";

(*b*) That it is always advisable to provide steel reinforcement in the footing;

(*c*) That steel columns should always be suitably riveted to a proper grillage in the mat or be of proper form at the base to make full use of the concrete mat. (Anchor-bolts in the ordinary sense of the word are not sufficient.)

On page 48,† the author states that "a better design, however, is furnished by making the floor in the form of a thick reinforced concrete slab. This not only produces a vibration-absorbing mat at the most efficient location, but also provides a good insurance against horizontal disturbance of the alignment of the unit." While recognizing the value of this "vibration-absorbing mat", why not also consider the full, proper use of the lower "mat". It may also be utilized as a "vibration-absorbing mat", or, perhaps more accurately, it might be termed a "vibration-lessening mat" when properly designed. None of the diagrams, tables, or curves in the paper indicates that consideration extended below the top of the "mat". In speaking of the pedestals on page 43†, the author states, "* * * it is believed that all turbo-generator manufacturers concur in the desire that they be made very rigid." The speaker's plea for the full scientific use of the "mat" is made because such use will give the most rigid pedestal. By proper design of the "mat", the axes of the columns can be kept vertical just above the mat, which is equivalent to reducing H and L in Fig. 3,‡ and with the same pedestal and supported weight, the frame will have a shorter natural period of vibration, which is a desirable result. This helps in a manner similar to that of lowering of the center of gravity "by hanging the condenser from the top of the pedestal". The author has stated that "the more rigid a pedestal is made, the shorter its natural period will be, and its members should be arranged so as to secure this end." The speaker claims the "mat" is a very important "member" of the most scientifically designed foundation for the support of a turbo-generator.

F. A. ERICSON,§ M. AM. SOC. C. E.—During the speakers connection with the New York Edison Company, and for the last few years with Thomas E. Murray, Incorporated, a variety of turbine foundations have been designed,

* Newark, N. J.

† *Proceedings*, Am. Soc. C. E., January, 1923.

‡ *Proceedings*, Am. Soc. C. E., January, 1923, p. 43.

§ Structural Engr., Thomas E. Murray, Inc., New York City.

the capacity of the machines ranging from 7 500 to 50 000 kw., in all 558 000 kw.

The steel design has been adopted for the following reasons:

1.—It allows more room for piping and auxiliaries and, therefore, enables a greater capacity of a given space or building to be obtained, thereby reducing the cost per kilowatt.

2.—It is believed that a better, more serviceable, and more durable foundation can be built of steel than of either plain or reinforced concrete.

3.—In some cases, the space is so limited as to exclude all possibility of using any other type.

The design adopted by the speaker's Company consists of two main longitudinal girders, connected by a system of deep and stiff cross-girders and auxiliary longitudinal girders where they were necessary, in order to provide sufficient support for the bed-plate of the turbo-generator. The columns are placed directly under these longitudinal girders, with bracing inserted wherever practicable both longitudinally and cross-wise. All the girders are provided with full-depth heavy angle connections to each other, and all connections to columns and bracing are stiff and substantial. Wherever the girders are in clusters, they are held together with bolts and gas-pipe separators, and the space between them is filled with concrete. This increases the lateral stiffness of the foundation to a great degree and also adds considerable weight, each of which tends to reduce vibrations. In designing the girders, a very low fiber stress of 5 000 to 6 000 lb. per sq. in. is used in order to minimize vibrations, and the allowable shear is reduced in proportion. Although this low fiber stress is used, nevertheless it is found necessary to increase the material in most of the girders, especially on the larger units, otherwise, the deflections would become excessive. A force proportional to the turbine load is assumed as acting in horizontal directions and is used in designing columns and bracing, but the bracing is always made extra heavy and direct acting wherever possible. All the details are designed with the idea of making a monolithic and continuous structure.

In order to avoid the possibility of unequal settlement of columns, the turbine foundation is set on a thick mat of continuous reinforced concrete, which is usually a part of the foundation for the operating room.

James H. Richardson,* Assoc. M. Am. Soc. C. E.—The speaker has had more or less to do with the designing of turbo-generator supports and wishes to present a friendly plea for the structural man as against the machine designer. The latter usually precedes the structural designer in planning power-plant units, arranging the condensers, ducts, and ramifications of pipes, large and small, often with too little regard for the structural parts of the unit.

If the designer of the supports could rely on the strength of curved columns and sinuous girders, all might be well. As this is not the case, "let the mechanical designer allow plenty of space for the most favorable arrangement

* New York City.

of the structural members necessary for rigidly supporting the turbo-generator unit."

F. M. Beer,* Assoc. M. Am. Soc. C. E.—Although not thoroughly conversant with turbo-generator practice, this paper brings to mind two installations, one of which is of 1 500-kv-a. and the other of 3 000-kv-a. rating, that are in use in an electric light and power plant in New York City. The supports of these units seem quite well to overcome the objectional vibrations set up while the machines are operating.

Two concrete walls of uniform thickness, parallel to the axes of the units, support transverse steel beams that carry the units. This arrangement appears best to meet the requirements of this class of equipment, as it provides the mass necessary to absorb vibration, where it is most effective and may be adapted to any size of unit. Access through these walls may be made wherever convenient and rigidity maintained even if the walls are chased or pierced to accommodate auxiliaries or piping.

* Cons. Engr., New York City.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

HENRY PERCY BORDEN, M. Am. Soc. C. E.*

DIED OCTOBER 19TH, 1922.

Henry Percy Borden was born at Port La Tour, Nova Scotia, on December 8th, 1872. He attended Mount Allison University, at Sackville, New Brunswick, and, later, McGill University, at Montreal, Que., Canada, having been graduated in Civil Engineering from the Faculty of Applied Science of that institution in 1902.

Following his graduation, Mr. Borden entered the Engineering Department of the Canadian Pacific Railway, where he was engaged on the design of steel bridges and masonry and the inspection of the fabrication and erection of steel bridges, until 1904. He then accepted a position with the Montreal Locomotive and Machine Company as Assistant Engineer, in the Structural Department, on estimating and design, and, later, as Assistant to the Chief Engineer in charge of the Estimating and Sales Department.

In 1906, Mr. Borden returned to the Canadian Pacific Railway as Architectural Engineer, in which capacity he had charge of the steel and reinforced concrete design for the Company's buildings, under the Chief Architect.

In 1908, he was appointed Secretary and Assistant Engineer to the Board of Engineers of the Quebec Bridge, which position he held until 1915, when he was made Assistant to the Chief Engineer of the same Board, and had charge of the designing office, mill, shop, and field inspection, under the Chief Engineer.

After the death, in 1916, of Charles C. Schneider, Past-President, Am. Soc. C. E., one of the members of the Board, Mr. Borden was appointed to fill the vacancy thus created, which position he retained until the successful completion of the bridge in 1918.

Following the completion of the Quebec Bridge, Mr. Borden moved to Ottawa, Ont., Canada, entering the office of the Consulting Engineer to the Dominion Government, as Bridge Engineer. He retained this position until 1921, when he opened an office in Ottawa and began private practice as Consulting Engineer, specializing in bridges and structures of steel and reinforced concrete.

Mr. Borden was married in 1906 to Edith Eva Hall, of L'Orignal, Ont., Canada, who, with two daughters, survives him.

He was a member of the Engineering Institute of Canada, the Country Club of Ottawa, the Royal Ottawa Golf Club, and the Beaconsfield Golf Club, of Montreal.

Mr. Borden was elected a Member of the American Society of Civil Engineers on May 13th, 1918.

* Memoir prepared by C. N. Monsarrat, M. Am. Soc. C. E.

WILLIAM CUSHING EDES, M. Am. Soc, C. E.*

DIED MAY 25TH, 1922.

William Cushing Edes, the son of Dr. Richard Edes, a noted Unitarian minister, and Mary Cushing Edes, was born in Bolton, Mass., on January 14th, 1856. He was educated at the Massachusetts Institute of Technology and was graduated therefrom in 1875 with the degree of B. S. in Civil Engineering.

Immediately after his graduation, Mr. Edes was engaged in statistical work for the United States Census Bureau. In 1877, he moved to San Francisco, Calif., and entered the service of the Spring Valley Water Company.

In 1878, he became connected with the Engineering Department of the Southern Pacific Company and was employed as Assistant Engineer, until 1882, on the location of that Company's line through Arizona, New Mexico, and Texas. The experience gained during these years on a pioneer undertaking of such magnitude, and a natural aptitude for this class of engineering work, determined the course of his professional career, and practically the whole of his life after the completion of the survey was devoted to railroad location and construction.

In 1883, Mr. Edes returned to Boston, Mass., and was engaged in private practice until 1886, in which year he again went to California and re-entered the service of the Southern Pacific Company as Assistant Engineer, continuing with that Company until 1896. During this time, he was in charge of location and construction work for the Company, the most important of which was the location of the Coast Route in California, from San Luis Obispo to Ellwood, a distance of 107 miles.

In 1896, Mr. Edes was appointed Principal Assistant Engineer of the San Francisco and San Joaquin Valley Railway Company, which was building a line from San Francisco to Bakersfield, Calif., a distance of 381 miles. He was in immediate charge of both the location and construction of this line and personally made the location of that part of the road between Stockton and Richmond, Calif., 69 miles, the most difficult section. This road was later acquired by the Atchison, Topeka, and Santa Fé Railway Company and is now a part of its Coast Lines. In his capacity of Principal Assistant Engineer, Mr. Edes was thrown into contact with the people and the public officials of the districts traversed by the railroad, and his kindly personality, tact, and honesty contributed largely to overcoming the many difficulties encountered on projects of this kind and to facilitating the progress of the work.

In 1900, he returned to the Southern Pacific Company and was engaged, until 1905, on difficult location for the reconstruction of various parts of that Company's lines in California, Nevada, and Utah, chief among which was the section between Rocklin, at the western base of the Sierra Nevada Mountains, and Truckee, on the eastern slope. In 1905, he was appointed District Engineer of Maintenance of Way for the lines of the Southern Pacific

* Memoir prepared by the following Committee of the San Francisco Section: Bernard Benfield, Jerome Newman, and Frank Thompson Oakley, Members, Am. Soc. C. E.

Company in the northern part of California, filling that position for two years.

The Southern Pacific Company and the Atchison, Topeka, and Santa Fé Railway Company, in 1907, acquired jointly the Northwestern Pacific Railroad Company, which owned a line extending northerly from San Francisco Bay to Willits, Calif., 133 miles, and southerly from Humboldt Bay to Shively, Calif., 38 miles. It was decided to complete the gap between Willits and Shively, 106 miles, and Mr. Edes was selected as Chief Engineer to carry out the task of making the location and directing the construction of this link connecting the separated sections of the road. The line ran through the canyon of the Eel River, along precipitous banks of unstable material, and Mr. Edes' location through this country, one of the most difficult pieces of railway work on the Pacific Coast, is an enduring monument to his skill.

On the completion of this work, Mr. Edes was appointed by President Wilson as Chief Engineer and Chairman of the Alaskan Engineering Commission, which had been created to take charge of the location and construction of the Government railroads in Alaska. Immediately after his appointment in May, 1914, Mr. Edes left for Alaska and, in April, 1915, sufficient progress had been made to recommend the adoption of the route by way of the Alaska Northern Railroad from Seward to Kern Creek, 71 miles, and thence by the construction of 400 miles of new road to Fairbanks. With the exception of timber suitable only for piling, ties, and small dimension lumber, there was nothing in the country with which to begin work; all construction material and equipment had to be brought in on steamers and facilities established for handling, storage, and repairs. The site of Anchorage, which was selected as the headquarters of the Commission, was practically uninhabited, and it was necessary to build a city at this place for the construction forces and towns at several other points for the development of the country and the administration of the innumerable activities connected with the railroad work. The country through which the road was to pass was an unbroken wilderness, entirely devoid of transportation facilities, and little was known about it. Notwithstanding these almost insuperable obstacles, at the end of 1915, there had been completed 35 miles of grading and 13 miles of track, as well as a considerable mileage of wagon roads.

In addition to the difficulties offered by the country, there was the added one of obtaining funds with which to prosecute the work vigorously. The United States was engaged in the World War and Congress was not in a mood to furnish money for other than war purposes; it was necessary, therefore, for Mr. Edes to spend a large part of his time in Washington, D. C., in order to secure appropriations for continuing the work in Alaska. One of the Commissioners had been appointed Governor of the Territory, the other had re-entered the Army, and Mr. Edes was left as the only remaining member, in full charge and responsible for all the activities of the project. The duties were most arduous, and it is a high tribute to his honesty and ability that he succeeded in convincing Congress that funds for continuing the work should be provided, although the Nation was carrying on a war which taxed its power and resources. The strain and worry under which he labored, however, had

seriously affected his health, which was never robust, and as the general problems had been solved and the work carried well along toward completion, he resigned from the Commission in 1920, in order to take up work which would be less exacting. He was engaged after that time in consulting practice, advising especially on matters pertaining to railroad projects. It was while returning by train to San Francisco from an inspection of a project on which he had been consulted, that he was stricken with heart failure, resulting in his death.

Mr. Edes was a man who won the respect and affection of all those associated with him. In the words of one of his old friends:

"He was honest—with him honesty was an instinct. He thought along honest lines, and by honesty I refer not so much to money matters as to all the varied elements which occur in the life of an engineer. He was honest to those above him in giving them the benefit of his thought even though it might be at variance with the popular conclusions; he was honest with those working for him, and he was honest with himself in that he was careful to do what he thought was right.

"Mr. Edes was a modest man—if one might suggest it, too modest for his own good in this rushing, driving world. One would never learn from him of his achievements and many were disposed to under-rate him because he was so self-effacing; but those who knew him well recognized the quality of the man and loved him for his gentleness and genuineness.

"The work he has done will live after him as a mark of his ability, and while, as time goes on, his name in connection therewith may be lost, yet the work itself will stand and be recognized as well designed and well done."

Mr. Edes was married on January 31st, 1901, to Miss Mary Burnham, of Oakland, Calif., who, with three brothers, Robert, Francis and John Edes, and a sister, Mrs. Cyrus A. Roy, survives him.

He took an active part in public and charitable affairs, and was particularly interested in Red Cross work.

Mr. Edes was elected a Junior of the American Society of Civil Engineers on September 1st, 1886, and a Member on November 4th, 1896.

WILLIAM DAVID UHLER, M. Am. Soc. C. E.*

Died October 27th, 1922.

William David Uhler, the son of the late Andrew Jackson Uhler and Susan E. Seiple Uhler, was born at Nazareth, Pa., on November 8th, 1872. He received his preparatory education in the public schools of Easton, Pa., having been graduated from the High School in June, 1889. In June, 1910, the Maryland Agricultural College, now the Maryland State University, conferred on him the honorary degree of Civil Engineer, in recognition of his work with the Maryland State Roads Commission.

During summer vacations, and prior to his graduation from High School, Mr. Uhler was employed on Corps work with the Lehigh Valley Railroad Company, and on the completion of his High School course, he entered the service

* Memoir prepared by H. E. Hilts, Assoc. M. Am. Soc. C. E.

of this Company, under Mr. I. W. Troxel, as Chainman, Rodman, and Instrumentman. In 1892, he became Assistant Engineer of the Baltimore and Lehigh Railroad Company, under Mr. Troxel, at Baltimore, Md., but returned to the Lehigh Valley Railroad Company in 1893 as Transitman, remaining with that Company until 1895. During these periods, he was employed on preliminary and location surveys and on construction work.

In 1895, Mr. Uhler returned to Baltimore as Assistant Chief Engineer of the Delaware, Maryland and Virginia Railroad Company (the Queen Anne Railroad), in which capacity he was employed on the preliminary and location surveys and the construction of this line. He retained this position until the road was ready for operation in 1897, and from 1897 to 1904, he served the Company as Assistant General Manager. His ability as an Engineer and an organizer was early recognized in his association with this Company, and his superior officer states that, in the later years of his services with this line, "he had full charge of all the Company's affairs."

In 1904, he was made County Road Engineer of Caroline County, Maryland, in charge of the construction and maintenance of highways and bridges. During his incumbency of this office, which was in the earlier days of highway development, he originated methods of design and maintenance, and under his tutelage and able executive direction an organization was developed, which gradually placed this work under business management and administration.

In 1908, under W. W. Crosby, M. Am. Soc. C. E., then Chief Engineer of the Maryland State Roads Commission, Mr. Uhler accepted the position of Assistant Engineer of the Maryland State Roads Commission. He remained with the Commission from 1908 to 1912, having had charge of survey work, the preparation of plans, and the details of the maintenance of the highways of that State. During this time, he organized the maintenance forces throughout the State, put into effect the patrol system of maintenance, and most effectually performed all these duties, in addition to having charge of all surveys and all drafting, and similar work, under the Chief Engineer. During this period, in the words of his Chief, "his brilliant mind, laudable ambition, unflagging energy, steadfast loyalty, and untiring application to duty, enabled him to succeed where many others would have failed, and his pleasing personality endeared him to all. Unquestionably, the present high standing, among their users, of the State roads of Maryland to-day, is due to the working out of the details of this maintenance by William D. Uhler."

In 1912, Mr. Uhler was recommended by Mr. Crosby for the position of Principal Assistant Engineer of the Bureau of Highways of Philadelphia, Pa., under the Blankenburg Administration. Mr. Uhler was the successful candidate by Civil Service examination for this post. From 1912 to 1915, he was engaged in the design, construction, and maintenance of city streets and of special park work. The then Director of Public Works, Mr. Morris L. Cooke, states that during this time:

"There was not anything that went on in the Bureau to which he did not make a real contribution. It was due, in a large measure to his capacity for detail, his indefatigableness, and his painstaking regard for the matters that must accompany public work, that we were able to accomplish as much as we

did in transferring the Department from the point where we employed one engineer out of a total of upward of a thousand employes, to where all of its work practically was carried on under engineering methods and for the most part by engineers. He had the capacity for telling citizens and others what was going to happen, to do it promptly, and in a manner which was inspiring in the confidence of the public."

In 1915, Governor Martin G. Brumbaugh appointed Mr. Uhler as Chief Engineer of the Pennsylvania State Highway Department, and from the date of his appointment to January 24th, 1918, when he was granted a leave of absence, his experience, energy, resourcefulness, and executive capacity were engaged in standardizing the maintenance and construction of the highways of the Commonwealth, and perfecting the nucleus of an organization which was to carry on during the World War.

On January 24th, 1918, Mr. Uhler was granted a leave of absence by Governor Brumbaugh, and was commissioned a Major in the Quartermaster Corps, Motors Division, U. S. Military Service. On May 8th, 1918, he was promoted to the rank of Lieutenant Colonel. He was discharged from the service on November 23d, 1918, and on March 29th, 1920, he accepted a commission as Colonel in the Quartermaster Section, Reserve Corps. George W. Goethals, Maj.-Gen., U. S. A. (*Retired*), M. Am. Soc. C. E., states of Colonel Uhler's activities during this period as follows:

"In January, 1918, when I took charge of all transportation for the War Department, it was decided to move supplies by motor trucks, where this could be done to advantage. This brought to the fore the necessity for the repair of old, and the construction of new, highways, and in connection with this work, I drafted the services of Colonel Uhler. This work was then expanded to determine whether new construction desired by States or municipalities would arise in war activities, since highway construction was practically stopped during the war period. All questions of this kind, as they concerned the War Department in the deliberations of the United States Highway Council, were passed upon by Colonel Uhler, who was thorough in his investigations and development of the facts and whose conclusions showed his ability and foresight. The soundness of his judgment in my earlier association with him soon led me to accept his recommendations without question. He was thoroughly reliable and efficient, performing a difficult task faithfully, conscientiously, and zealously."

On being honorably discharged from the service, Colonel Uhler returned to his work as Chief Engineer of the Pennsylvania State Highway Department, and was re-appointed Chief Engineer by Governor William C. Sproul on April 7th, 1920. In association with the late Lewis S. Sadler, State Highway Commissioner, and the Assistant Commissioner, George H. Biles, M. Am. Soc. C. E., Colonel Uhler created and enlarged the State Highway Department and carried on one of the largest post-war programs of State highway construction. Mr. Biles' tribute at the time of Colonel Uhler's death best exemplifies the thoughts of Colonel Uhler's engineering organization:

"The death of Colonel William D. Uhler is an irreparable loss to the State of Pennsylvania. From my many years of association with him I am convinced that he had no peer among highway engineers in this country. In the vast road building program inaugurated by Governor Sproul, he performed such conspicuous service in engineering skill that Pennsylvania to-day leads

the nation in the construction of modern highways. His life was dedicated to his ideals in road building to the exclusion of all other things. This intense application soon manifested itself in a weakened physical condition, which he could not overcome, and finally caused his untimely end. I am grieved beyond expression by the loss of such an able and conscientious colleague, and the passing of a sterling character and true friend."

His peculiar fitness for his work may be readily summarized in the words of Thomas H. MacDonald, Chief of the Bureau of Public Roads, United States Department of Agriculture:

"Colonel Uhler was an outstanding figure in the field of Highway Engineering and Administration. His influence was national. He was a leader in the forming of policies and legislation, State and National, to provide adequate modern highways. The accomplishment in engineering of the State Highway Department is a tribute to his leadership. As an organization, the Bureau of Public Roads has respected and appreciated his courage, ability, and integrity of conduct and purpose in his administration of a great public trust. As individuals, we feel most keenly the loss of a real friend."

Colonel Uhler was a member of the American Society of Municipal Improvements, the American Concrete Institute, and the American Society for Testing Materials. He was also a member and a Director of the American Road Builders' Association, and a member of the International Association of Road Congresses and the Engineers' Club of Philadelphia. He was a Past-President and a Director of the American Association of State Highway Officials, having served first as Chairman of its Executive Committee in 1917. He was elected President in 1918, and was re-elected a member of the Executive Committee and served as such until his death. He was also Chairman of the Committee on Standards, and a member of the Committee on Motor Truck Regulation.

Colonel Uhler was married on December 31st, 1896, to Marguerite Stevens, daughter of the late Mr. and Mrs. B. S. Stevens, of Denton, Md., who survives him, together with his mother and three sisters.

He was a member of Temple Lodge, F. and A. M., of Denton, Md., Knight Templar Commandery of Easton, Md., Harrisburg Consistory, A. A. S. R., and Boumi Temple, A. A. O. N. M. S., of Baltimore, Md.

Colonel Uhler was elected an Associate Member of the American Society of Civil Engineers on May 6th, 1908, and a Member on March 1st, 1910.

ARCHIE LEE HARRIS, Assoc. M. Am. Soc. C. E.*

DIED JULY 17TH, 1922.

Archie Lee Harris, the son of Lorenzo and Isabel (Combs) Harris, was born at Orange, Mass., on February 17th, 1872. His early education was received in the public schools of his home town where he taught for a number of years after his graduation from the High School. He then entered the University of Michigan, from which he was graduated in 1898, receiving the degree of Bachelor of Science in Civil Engineering.

* Memoir prepared by M. R. Kays, Assoc. M. Am. Soc. C. E.

His first position after graduation was that of Instrumentman and Draftsman with the United States Board of Engineers on deep waterway surveys. During 1900 and 1901, Mr. Harris was Draftsman on the New York State Barge Canal, at Albany, N. Y., and with the U. S. Isthmian Canal Commission, at Washington, D. C. Prior to 1903, he was also employed as Draftsman by the Russell Wheel and Foundry Company, of Detroit, Mich., the Hamilton Bridge Works, Limited, of Hamilton, Ont., Canada, and the East River Division of the Pennsylvania, New York and Long Island Railroad Company.

In May, 1903, soon after its organization as a separate bureau, Mr. Harris was appointed Assistant Engineer by the United States Reclamation Service and assigned to the Salt River Project at Phœnix, Ariz. During the next seven years, he held the positions of Draftsman, Chief Draftsman, and Designer in the Field Office at Roosevelt, Ariz. (Roosevelt Dam), which, during the early years of construction, was the principal engineering office for the Project. During this time, he had a responsible part in the design and construction of the larger structures on the Project, but there are two, namely, the diversion dam at the head of the Roosevelt Power Canal and the Granite Reef Diversion Dam, the largest river structure below the reservoir, for the design of which he was largely responsible and is entitled to especial credit. Incidental to the design of these dams, Mr. Harris made some original investigations and studies, in an effort to improve on the lifting devices for the outlet gates which, as previously designed, were operated with great difficulty. After conducting a series of experiments on full-sized gates under actual field conditions, he found that the coefficient of friction which had been used to that time was entirely too low. By the use of new coefficients, he was able to design operating mechanisms which gave complete satisfaction, resulting in the adoption of new standards by the Reclamation Service. He thus came to be recognized as an expert in this work and designed practically all the gates built on the Project thereafter.

About the time the Roosevelt Dam was completed, and near the completion of his period of service for the Government, Mr. Harris was the victim of an accident which nearly resulted fatally. While engaged in an inspection of the outlets and tunnel at the bottom of the dam, with water at a high elevation in the reservoir above, the gates were opened suddenly by mistake and he and a companion were caught in a rushing torrent of water and swept through several hundred feet of tunnel into the river channel below. His companion was drowned, and Mr. Harris escaped only by the exercise of extraordinary presence of mind and great physical effort.

Mr. Harris resigned from the Reclamation Service in 1910 and opened an engineering office in Phœnix, specializing in irrigation. He designed and laid out a number of private irrigation systems and also laid out the deep-well pumping and distribution system for the 12 000-acre, long staple cotton plantation of the Goodyear Tire and Rubber Company, at Litchfield, Ariz. In 1915, he moved to Los Angeles, Calif., where he opened a consulting office, continuing, however, to serve several of his Arizona clients whose work occupied much of his time until his death. His principal work, outside of irrigation,

was the design and construction of a reinforced concrete pleasure pier at Manhattan Beach, Calif. At the time of his death, Mr. Harris was Consulting Engineer for a number of proposed irrigation and power projects in Arizona, including the 40 000-acre project at Beardsley, near Phœnix, for which he had directed all the engineering work since its inception. He was also employed, in the capacity of Chief Engineer, by the Paradise-Verde Irrigation District, at Phœnix, for which he was completing final plans and estimates for a system to irrigate from 80 000 to 100 000 acres of land adjacent to the present Salt River Project. As planned by him, the system was to include several large storage and diversion dams on the Verde River and other streams, one or more hydro-electric power plants, and a modern, concrete-lined, distribution system.

Mr. Harris was married on December 25th, 1911, to Miss May Alderman who, with a daughter, Louise, survives him.

His untimely death cut short a most promising professional career and prevented accomplishments for which he was especially well fitted by training and years of experience and observation. Nevertheless, he had become recognized as an authority on irrigation, in the Southwest, and especially in Arizona, where he was intimately familiar with conditions. He was an enthusiastic believer in reclamation, but it was a conservative enthusiasm resulting from a thorough and conscientious study of the many matters which must be taken into account in laying the foundation for successful development. He was a builder and asked for no greater reward than to have a part in the constructive work of controlling and utilizing the wasted water resources with the resulting permanent contribution to the wealth and producing power of the nation. He was a high type of engineer, active and resourceful in the field, painstaking and thorough in investigation and analysis, conservative in his advice and recommendations, considerate of associates and subordinates and thoroughly loyal to the interests of employer or client.

Mr. Harris' rating as an engineer was equalled only by his standing and reputation as a man. Just as he set up as a shrine and ever kept in view and observed a rigorous code of professional ethics and the highest standard of service, so did he adopt and govern himself by the highest ideals of personal conduct and citizenship and live and practice them in his daily life. Brought up in the Universalist faith, although not in later years an active member of any denomination, he was a true Christian gentleman, because he not only believed in the rules laid down for guidance in the Good Book, but he practiced them in his work, in his dealings with his fellow men, and in his home. Although he was affable, companionable, and entertaining, with a normal sense of humor, he was given to reflection and the study of serious things and enjoyed discussions of the political, economic, and moral questions of the day. His interest in the worth-while things which with him were favorite topics of conversation, made him a delightful companion. Above all, however, he was a gentleman—clean, moral, honest, and upright in every respect—and he will be remembered by those who knew him well, for his genuine friendliness, his even disposition, his unselfishness, his respect for the rights and views of others, and his high-minded conception of service. "Honorable and high-minded" and

"a very decided loss to the community and to the Engineering Profession", are typical of the testimonials volunteered by distinguished professional associates who knew him best.

He was the author of a paper entitled, "The Diversion of Irrigating Water from Arizona Streams", which was published by the Society.* He was a member of the American Association of Engineers.

Mr. Harris was elected an Associate Member of the American Society of Civil Engineers on May 31st, 1910, and was an active member of the Los Angeles Section of the Society.

ARTHUR FRANCIS HOLLAND, Assoc. M. Am. Soc. C. E.†

DIED JUNE 22D, 1922.

Arthur Francis Holland, the son of Edwin and Margaret Holland, was born at Barre, Vt., on September 10th, 1888. He received his early education in the public schools of Seattle, Wash., to which city his parents had moved. He afterward entered Norwich University and was graduated therefrom in 1913 with the degree of Bachelor of Science in Civil Engineering.

In July, 1913, immediately after leaving college, Mr. Holland entered the United States Engineer Department at Wheeling, W. Va., and remained in that service until September, 1915. During this time, he served as Inspector and had charge of construction work on the Ohio River locks and dams in the Wheeling District. He was also engaged on the design, estimates, drafting, and office computations and records, in connection with these structures.

In September, 1915, he entered the employ of The Koppers Company, as Instrumentman, on the by-product coke plant then being constructed for the River Furnace Company, at Cleveland, Ohio. In March, 1916, he was transferred to the plant being built for the Seaboard By-Product Coke Company, at Jersey City, N. J., as Field Engineer, in which capacity he had charge of the Engineer Corps and all layout work in connection with the plant, until August, 1917.

In September, 1917, Mr. Holland entered the Engineer Reserve Officers' Training Camp at Camp American University, Washington, D. C., and was assigned to Company 2. On the completion of the course, no openings were available in active service, and he returned from camp in November, although he was not discharged from the service until sometime during 1918.

In December, 1917, he returned to the service of The Koppers Company and was placed in charge of the construction of a large by-product coke oven plant for the Providence Gas Company, at Providence, R. I. This work was completed in June, 1919. From July, 1919, to February, 1920, Mr. Holland was in charge of the tearing down and rebuilding of a battery of coke ovens for the Inland Steel Company, at Indiana Harbor, Ind. During the summer of 1920, his health did not permit him to engage in active work, although part

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 932.

† Memoir prepared by D. M. Craig, Esq., Pittsburgh, Pa.

of this time was spent in the Pittsburgh Office of The Koppers Company. In November, 1920, he was placed in charge of the work of dismantling and rebuilding a coke oven battery for the Milwaukee Coke and Gas Company, at Milwaukee, Wis., and it was while he was thus engaged that he was overcome by the illness which resulted in his death on June 22d, 1922.

Mr. Holland was a man of unusual character and integrity, loyal to his associates, conscientious and painstaking in his work. His own comfort and convenience were never permitted to interfere with his duty to his employer, and he was kind, and considerate to and well beloved by his subordinates.

He was made a Master Mason on June 1st, 1920, and was a member of Wetzel Lodge No. 39, F. and A. M. He is survived by his wife, who was Quintie Louise Potts, and two sons, John Arthur and James Edward.

Mr. Holland was elected an Associate Member of the American Society of Civil Engineers on October 14th, 1919.

SPENCER BAIRD NEWBERRY, Assoc. M. Am. Soc. C. E.*

Died November 28th, 1922.

Spencer Baird Newberry, the third son of Professor John Strong Newberry, sometime State Geologist of Ohio and, for many years, Professor of Geology at Columbia University, New York City, was born in Cleveland, Ohio, on May 11th, 1857, and was graduated at the School of Mines, Columbia University, in 1878, taking his doctor's degree there one year later. He studied in Berlin, Germany, and Paris, France, for two years, and, from 1881 to 1892, was Acting Professor of Chemistry at Cornell University, Ithaca, N. Y. In 1889, he was sent by the Government as Expert Commissioner to the Paris Exposition, and, in 1893, he was a member of the Jury of Awards at the World's Fair at Chicago, Ill.

While at Cornell University, Dr. Newberry became interested in the manufacture of Portland cement, then in its infancy in the United States. Recognizing the possibilities for the future development of the cement industry, he left Cornell and founded the Sandusky Portland Cement Company with its first plant at Bay Bridge, near Sandusky, Ohio, built in 1892. He was General Manager of this Company until 1912, when he became President, which position he occupied until his death. The business developed rapidly and other plants were built in Syracuse, Ind., Dixon, Ill., and York, Pa., which have been successfully operated for many years. During the last year of his life, Dr. Newberry's energies were devoted to the planning and erection of a new plant near Toledo, Ohio, on a valuable and unique deposit of limestone discovered through a paragraph in his father's first report on the Geological Survey of Ohio in 1874, and the plant, incorporating as it does the results of his wide experience in the art, is expected to be the last word in modern cement practice. Although he did not live to see it in operation, he took the greatest satisfaction in its construction.

* Memoir prepared by W. B. Newberry, Esq., Cleveland, Ohio.

Dr. Newberry was the first to demonstrate that white Portland cement could be made successfully, and the factory at York, Pa., is devoted to the manufacture of this product alone, which has found wide use in ornamental architectural work. He was also successful in inventing and manufacturing an integral water-proofing compound for concrete, and had taken out many patents on chemical processes in the cement and other industries. His early investigations into the chemical constitution of cement and the determination of a theoretical formula for proportioning the raw materials have been recognized as having marked an epoch in the industry, and he was conceded to be one of the foremost authorities on the chemistry of cement as well as on the mechanical processes of its manufacture. He was also the author of many articles and monographs on the art, which are considered highly important and valuable contributions to the knowledge of this industry.

Dr. Newberry was married at Ithaca, N. Y., in 1882, to Clara, daughter of the Hon. Andrew D. White, then President of Cornell University. She died in 1907, leaving two sons, Andrew W. and Arthur C. Newberry, both of whom survive their father. In 1908, he was married to Helene Printy, of Vickery, Ohio, who also survives him.

Dr. Newberry was elected an Associate Member of the American Society of Civil Engineers on January 6th, 1897.

WINSLOW BARNES WATSON, Assoc. M. Am. Soc. C. E.*

Died August 6th, 1922.

Winslow Barnes Watson, the eldest son of the late Judge Winslow Charles Watson and Ella Barnes Watson, was born in Plattsburg, N. Y., on August 28th, 1880. He was educated in the Plattsburg schools, having been graduated from the High School in 1898. It was the desire of Judge Watson that his son should be a lawyer. For that reason, he studied law in his father's office during two years following his graduation from High School. In the fall of 1900, he entered Union College from which he was graduated in 1904, with the degree of Bachelor of Engineering.

Immediately after finishing college in 1904, Mr. Watson was appointed a Computer and Instrumentman on the construction of the New York State Barge Canal. He remained in the employ of the State of New York until 1907, when he went to Panama where he was engaged on the construction of the Canal, and where he remained until 1908. At this stage of the construction of the Panama Canal, the sanitary conditions were not all that could be desired, and, in 1908, he again returned to the construction of the New York State Barge Canal.

From 1909 to 1912, Mr. Watson was in charge of Barge Canal Construction Contract No. 25, which represented one of the largest contracts in the State, and covered a distance of 13 miles of prism excavation, one lock, three dams, six bridges, and many other miscellaneous structures, approximating in cost

* Memoir prepared by E. W. Wendell, Assoc. M. Am. Soc. C. E.

nearly $2 000 000. In 1913 and 1914, he was in charge of all final estimates and final accounts for the contracted work on the Barge Canal covering the entire Eastern Division. Also, at this time, the surveys, designs, and estimates for the proposed Barge Canal Extension of the Glens Falls Feeder on the Champlain Canal were under his supervision, as well as practically all the miscellaneous surveys, designs, and constructions in the State Engineer's Department.

In 1915, Mr. Watson became associated with O'Connor and Chapman, Consulting Engineers, of Albany, N. Y. He remained with this firm until 1917, when he accepted the position of Engineer for the Holler, LaDu Corporation, of Fort Edward, N. Y.

Before the troops were called into service for the expedition against Mexico, in 1916, he had joined the 10th Regiment of the New York National Guard, and he went with that regiment as a Second Lieutenant. When the National Guard was mustered into Federal Service in 1917, as a component of the 106th Infantry, he was commissioned as First Lieutenant. He won his captaincy in France. That his military record in the World War was one to be proud of, can be easily seen by the following brief résumé: East Poperinghe Line, July 9th to August 20th, 1918; Dickebusche Sector, August 30th, 1918; LaSalle River, October 17th, 1918; DeMer Ridge, October 18th, 1918; and St. Maurice River, October 19th, 1918, which locations, and those dates, bring to the mind the fiercest fighting.

Captain Watson's service chevrons were authorized on November 23d, 1918, in Special Order 329, Headquarters 106th Infantry. His wound chevron was authorized on December 11th, 1918, by S. O. 106, Hdq. 106th Inf., Special Order No. 31, which reads as follows:

"Hdq. 27th Div. U. S. A.
"American E. F. France.
"Jan. 31, 1919.

"Capt. Winslow B. Watson,
"106 Infantry.
"Citation:
"For courage and determination in making personal reconnaissance under heavy enemy fire; this near St. Maurice River, October 20th, 1918."

On his return from France, and as soon as the 27th Division was mustered out of service, Captain Watson again associated himself with the Holler, LaDu Corporation, at Fort Edward, N. Y., and remained with this Corporation until 1920.

In 1920 and 1921, he had charge of a number of special departmental constructions in the Department of Public Works of the State of New York. In 1922, he was placed in charge of the construction of the new water supply project for the City of Plattsburg, in the capacity of Superintendent in Charge and Manager for Construction, and served in this capacity until his accidental death from drowning on August 6th, 1922.

Captain Watson's engineering experience extended over such a wide variety of work that his knowledge of construction was extensive, and he had just

reached an age when all this experience and knowledge could be utilized to its fullest extent.

He is survived by his wife, who before their marriage in 1907 was Miss Irene Signor, his daughters, Mary and Emily, his mother, Mrs. W. C. Watson, a sister, Miss Ellen Watson, and two brothers, Richard P. Watson and Mark S. Watson.

Captain Watson was elected an Associate Member of the American Society of Civil Engineers September 12th, 1916.